On the chatter frequencies of milling processes with runout

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Abstract

The detection of undesirable vibrations in milling operations is an important task for the manufacturing engineer. While monitoring the frequency spectra is usually an efficient approach for chatter detection, since these spectra typically have a clear and systematic structure, we show in this paper that the stability of the cutting process cannot always be determined from solely viewing the frequency spectra. Specifically, the disturbing effect of the tool runout can sometimes prevent the proper determination of stability. In this paper, we show these cases can be classified by alternative analysis of the vibration signal and the corresponding Poincaré section. Floquet theory for periodic systems is used to explore the influence of runout on the structure of milling chatter frequencies. Finally, the results from theoretical analysis are confirmed by a series of experimental cutting tests.

Keywords: Periodic system; Floquet theory; Machine tool vibrations; Chatter identification

1. Introduction

Machine tool chatter is still a problem for the machining community. These violent vibrations of the machine tool are problematic since they result in a poor surface finish, cause large-amplitude acoustic emissions, and can sometimes lead to tool failure. Therefore, it is highly important to detect the onset of these vibrations.

Stability lobe diagrams for machining processes were developed by Tobias [1] and Tlusty et al. [2] in the sixties and the corresponding theory for interrupted cutting process, such as milling, has much attention over the past decade starting with the work of Altintas and Budak [3,4]. Since then, several sophisticated models and analysis techniques have been developed to investigate the dynamic behavior of the milling process (see, e.g. [5–9]). More recent analytical investigations have led to the realization of a new instability behavior: in addition to quasi-periodic chatter (Hopf bifurcation), period doubling (period 2, flip bifurcation) has also been an observed instability behavior in milling [10–12].

Monitoring the vibration frequencies during machining is an efficient method for identifying machine tool chatter and distinguishing between different types of instabilities. In contrast to turning operations, which are characterized by a single chatter frequency, chatter vibrations in milling contain multiple frequencies due to the periodic nature of the process as it was shown in [13]. For ideal milling operations with symmetric tools, the cutting force is periodic at the tooth passing (TP) period. In this case, the vibration frequencies have a well defined, special structure, and stable and unstable machining cases can clearly be assessed based on the frequency spectra (see [13]). However, in practice, several other effects arise that influence and sometimes destroy the nice structure of the chatter frequencies. One of them is runout where the geometric axis of the milling cutter differs from the rotation axis. Runout causes the chip load to be distributed unevenly among the cutting teeth and shifts the frequency content of the cutting force signal away from the TP frequency and towards the spindle rotation (SR) frequency.
the modal mass, damping and stiffness matrices, and tip in the

where the vector $x(t)$ contains the displacement of the tool
tip in the $x$ and $y$ directions, the matrices $M$, $C$ and $K$ are
the modal mass, damping and stiffness matrices, and $a_n$ is
the axial depth of cut. The time delay is equal to the TP
period $\tau = \Omega/(60)N$, where $\Omega$ is the spindle speed in rpm
and $N$ is the number of teeth of the miller. The elements of
the directional force coefficient matrix $H(t)$ are

$$H_{xx}(t) = \sum_{j=1}^{N} \rho_j \theta(\varphi_j(t))(K_1 \cos \varphi_j(t)$$

$$+ K_n \sin \varphi_j(t)) \sin \varphi_j(t), \quad (2)$$

$$H_{xy}(t) = \sum_{j=1}^{N} \rho_j \theta(\varphi_j(t))(K_1 \cos \varphi_j(t)$$

$$+ K_n \sin \varphi_j(t)) \cos \varphi_j(t), \quad (3)$$

$$H_{yx}(t) = \sum_{j=1}^{N} \rho_j \theta(\varphi_j(t))(-K_1 \sin \varphi_j(t)$$

$$+ K_n \cos \varphi_j(t)) \sin \varphi_j(t), \quad (4)$$

$$H_{yy}(t) = \sum_{j=1}^{N} \rho_j \theta(\varphi_j(t))(-K_1 \sin \varphi_j(t)$$

$$+ K_n \cos \varphi_j(t)) \cos \varphi_j(t), \quad (5)$$

where $K_1$ and $K_n$ are the tangential and the normal cutting
force coefficients, and

$$\varphi_j(t) = \frac{2\pi \Omega}{60} \tau + \frac{2\pi(j-1)}{N} \quad (6)$$

is the angular position of the $j$th cutting edge. The function
$g(\varphi_j(t))$ is a screen function, it is equal to 1 if the $j$th tooth is
cutting or 0 when not cutting:

$$g(\varphi_j(t)) = \begin{cases} 1 & \text{if } \varphi_e < \varphi_j(t) < \varphi_a, \\ 0 & \text{otherwise}, \end{cases} \quad (7)$$

where $\varphi_e$ and $\varphi_a$ are the entry and exit angles of the cut.

The elements of the stationary cutting force vector $G(t)$ are

$$G_x(t) = a_x f_z H_{xx}(t), \quad (8)$$

$$G_y(t) = a_y f_z H_{yy}(t), \quad (9)$$

where $f_z = \nu T$ denotes the feed per tooth with $\nu$ being the
feed speed.

The runout for each tooth is modeled by a single factor
$\rho_j$, $j = 1, 2, \ldots, N$. If there is no runout (i.e.,
$\rho_j = 1$, $j = 1, 2, \ldots, N$), then the least period of $H(t)$ and
$G(t)$ is the TP period $\tau$. This case corresponds to the
conventional milling models (see, e.g., [3,4,20]).

In practical milling processes, runout cannot be avoided,
and $\rho_j \neq 1$, $j = 1, 2, \ldots, N$. In this case, the least period of
$H(t)$ and $G(t)$ is the SR period $T = 60/\Omega = N\tau$. Usually,
the discrepancies between the cutting forces acting on the
different teeth are small. This explains that a $\tau$-periodic
cutting force variation assumption is usually a good
approximation, in spite of the fact that the least period is
actually the SR period $T$.

Fig. 2 shows the variation of the cutting force acting on
the tool during machining with a two fluted miller ($N = 2$). It
can clearly be seen that the teeth experience different loads. The accompanied power spectral density (PSD)
diagram shows the additional frequency peaks at the multiples of the SR frequency (1/\(T\), 3/\(T\), etc.).

3. Stability analysis and vibration frequencies

Stability of the milling process can be estimated theoretically via the analysis of the governing Eq. (1). According to the Floquet theory of delayed differential equations [21], an infinite dimensional operator (monodromy operator) can be associated with the system that gives the connection between the current state of the cutter and the state one period earlier. This period is equal to the least period of the time-dependent coefficients in the equations and is often called principal period. Thus, for the milling model without runout, the principal period is equal to the TP period \(T\), while for the milling model with runout, the principal period is equal to the SR period \(T\). Stability properties are described by the eigenvalues of the monodromy operator, also called as characteristic multipliers or Floquet multipliers. If all the characteristic multipliers are in modulus less than 1, then the system is asymptotically stable. Since delayed systems usually have infinitely many characteristic multipliers, their stability calculation cannot be performed in closed form, but numerical approximations can be applied. Here, two methods are used, the semi-discretization method [22], and the time finite element method [23]. The basic point of both methods is that they provide a finite dimensional matrix approximation of the monodromy operator and the eigenvalues of this matrix can be determined numerically.

3.1. Vibration frequencies for the milling model with runout

In multi-flute machining, runout practically always arises since the tool is never ideally symmetric. In this case, the principal period is equal to the SR period \(T\). The chatter frequencies can be determined based on the Floquet theory of periodic systems [21]. Let \(\lambda_1\) denote the critical (maximal in modulus) characteristic multiplier of system (1), and let \(\xi_1\) denote the corresponding characteristic exponent, which satisfies \(\lambda_1 = e^{i\xi_1 T}\) with \(-\pi < \text{Im} \lambda_1 < \pi\). The corresponding solution of the system can be written in the form

\[
x(t) = p(t) e^{i\xi_1 t} + \bar{p}(t) e^{-i\xi_1 t},
\]

(10)

where \(p(t)\) is a \(T\)-periodic function, and bar denotes complex conjugation. Clearly, the system is just on the stability limit, if \(\text{Re} \lambda_1 = 0\), that is if \(\lambda_1 = i\omega_1\) with \(i\) denoting the imaginary unit. Fourier expansion of function \(p(t)\) gives

\[
p(t) = \sum_{k=-\infty}^{\infty} C_k e^{i(k\omega_1 T)t},
\]

(11)

where \(C_k\) are complex coefficient vectors. Substitution of Eq. (11) and \(\lambda_1 = i\omega_1\) into Eq. (10) results in

\[
x(t) = \sum_{k=-\infty}^{\infty} (C_k e^{i(k\omega_1 + k\Omega T)t} + \bar{C}_k e^{-i(k\omega_1 + k\Omega T)t}).
\]

(12)

The exponents in Eq. (12) give the frequency content of the motion in rad/s. The corresponding chatter frequencies in Hz are

\[
f_{\text{chatter}} = \pm \frac{\omega_1}{2\pi} + \frac{k}{T} = \pm \frac{\omega_1}{2\pi} + \frac{k\Omega}{60} \ [\text{Hz}]
\]

(13)

with \(k = 0, \pm 1, \pm 2, \ldots\). Of course, only the positive frequencies have physical meaning.
Note that $\omega_1$ equals the phase angle describing the direction of $\mu_1$ on the complex plane. Once the critical characteristic multiplier $\mu_1$ is determined numerically (using either the semi-discretization method [22] or the time finite element method [23]), then $\omega_1$ can be computed as

$$\omega_1 = \frac{1}{T} \text{Im}(\mu_1) = \frac{1}{T} \arctan\left(\frac{\text{Im} \mu_1}{\text{Re} \mu_1}\right)$$

(14)

with $-\pi < \omega_1 \leq \pi$.

Here, we can distinguish between three different cases:

1. **Quasi-periodic (QP) chatter**: The critical characteristic multiplier is a complex pair $(\mu_1, \mu_1)$ located on the unit circle: $|\mu_1| = 1$. This type of stability loss corresponds to the secondary Hopf (or Neimark–Sacker) bifurcation of periodic systems that is topologically equivalent to the Hopf bifurcation of autonomous systems. In this case, quasi-periodic vibrations arise during the loss of stability. The corresponding chatter frequencies are

$$f_{\text{QP}} = \pm \frac{\omega_1}{2\pi} + \frac{k}{\tau} = \pm \frac{\omega_1}{2\pi} + \frac{k\Omega}{60} \quad \text{[Hz]},$$

$$k = 0, \pm 1, \pm 2, \ldots$$

(15)

2. **Period 1 (P1) chatter**: The critical characteristic multiplier is $\mu_1 = 1$ and $\omega_1 = 0$. This case corresponds to the period 1 (or cyclic) fold bifurcation of periodic systems that is topologically equivalent to the saddle-node bifurcation of autonomous systems. The chatter frequencies are

$$f_{\text{P1}} = 0 + \frac{k}{\tau} = \frac{k\Omega}{60} \quad \text{[Hz]},$$

$$k = 0, \pm 1, \pm 2, \ldots$$

(16)

In this case, the chatter frequencies are equal to the multiples of the SR frequency.

3. **Period 2 (P2) chatter**: The critical characteristic multiplier is $\mu_1 = -1$ and $\omega_1 = \pi/\tau$. This case corresponds to the period 2 (or period doubling, or flip) bifurcation of periodic systems, and there is no topologically equivalent type of bifurcation for autonomous systems. The corresponding chatter frequencies are

$$f_{\text{P2}} = \frac{1}{2\tau} + \frac{k}{\tau} = \frac{\Omega}{120} + \frac{k\Omega}{60} \quad \text{[Hz]},$$

$$k = 0, \pm 1, \pm 2, \ldots$$

(17)

In this case, the chatter frequencies are equal to the multiples plus a half of the SR frequency.

Besides the chatter frequencies listed above, the SR frequency

$$f_{\text{SR}} = \frac{1}{T} = \frac{\Omega}{60} \quad \text{[Hz]}$$

(18)

and its harmonics also appear due to the $T$-periodic forcing $G(t)$. While the chatter frequencies in Eqs. (15)–(17) arise only for unstable machining, the harmonics of the SR frequency $f_{\text{SR}}$ show up both for stable and unstable processes.

It can be seen that the period 1 chatter frequencies coincide with the multiples of the SR frequency, so the period 1 case and the stable machining cannot be distinguished based on the frequency spectrum.

Note that the above classification of different types of chatter is somewhat different than that of the corresponding literature [11–13], where the TP period $\tau$ is considered as the principal period. In fact, for a tool with even number of cutting teeth, period two chatter (when viewed with respect to the TP period $\tau$) will lead to period one motions (when viewed with respect to the SR period $T$).

### 3.2. Vibration frequencies for the ideal milling model without runout

Milling operations without runout can practically be established by using a single fluted tool (see, e.g. [13,20]). In this case, the principal period is equal to the TP period $\tau = T/N$. The vibration frequencies can be determined similarly to (15)–(17) with substitution of $\tau$ instead of $T$:

$$f_{\text{QP-ID}} = \pm \frac{\omega_1}{2\pi} + \frac{k}{\tau} = \pm \frac{\omega_1}{2\pi} + \frac{k\Omega}{60} \quad \text{[Hz]},$$

$$k = 0, \pm 1, \pm 2, \ldots$$

(19)

$$f_{\text{P1-ID}} = 0 + \frac{k}{\tau} = \frac{k\Omega}{60} \quad \text{[Hz]}, \quad k = 0, \pm 1, \pm 2, \ldots$$

(20)

$$f_{\text{P2-ID}} = \frac{1}{2\tau} + \frac{k}{\tau} = \frac{\Omega}{120} + \frac{k\Omega}{60} \quad \text{[Hz]},$$

$$k = 0, \pm 1, \pm 2, \ldots$$

(21)

Here, index ID refers to “ideal” milling model (i.e., without runout). Besides the chatter frequencies in Eqs. (19)–(21), the TP frequency

$$f_{\text{TP}} = \frac{1}{\tau} = \frac{N\Omega}{60} \quad \text{[Hz]}$$

(22)

and its harmonics also appear, since the forcing $G(t)$ is now $\tau$-periodic.

It is known that for this zero runout model, only quasi-periodic chatter and period 2 chatter may occur, while period 1 chatter never arises. This can easily be seen: if $\mu_1 = 1$ is the critical characteristic multiplier, then the Floquet theory for the $\tau$-periodic system implies $x(t) = \mu_1 x(t - \tau)$. Consequently, $x(t) - x(t - \tau) = 0$. Substitution this to Eq. (1) yields

$$\mathbf{M} \ddot{x}(t) + \mathbf{C} \dot{x}(t) + \mathbf{K} x(t) = \mathbf{G}(t).$$

(23)

For positive definite matrices $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$, this system is always stable, consequently, it cannot experience stability loss if $\mu_1 = 1$. Therefore, chatter can be either quasi-periodic or period 2 that can be assessed clearly based on the frequency spectrum.
Note again that the period 2 chatter presents period 1 motion when viewed with respect to the SR period $T$ instead of the TP period $t$.

4. Zero runout vs. nonzero runout

In order to demonstrate the effect of runout, first, a theoretical study is presented. In Fig. 3, two stability charts and the corresponding chatter frequencies are presented for a 5% radial immersion down-milling operation with a two fluted cutting tool ($N = 2$). The parameters used in the computation are given in Section 5.

Stability charts were determined by both the semi-discretization method [22] and the time finite element method [23]. Both methods resulted exactly in the same stability boundaries, therefore they are not distinguished in Fig. 3.

4.1. Zero runout

Left panels in Fig. 3 correspond to the milling model without runout (i.e., $\rho_1 = \rho_2 = 1$). The principal period of the system is equal to the TP period $t$. Consequently, the chatter frequencies can be determined according to Eqs. (19)–(21). In this case only quasi-periodic chatter and period 2 chatter may arise, as it was shown by Eq. (23). Such cases are depicted by points A (quasi-periodic) and B (period 2). In the frequency diagram, dashed lines denote the TP frequency and its higher harmonics. The quasi-periodic chatter frequencies for point A are denoted by squares, the period 2 frequencies for point B are denoted by triangles. The corresponding locations of the characteristic multipliers are shown in the bottom of Fig. 3.

4.2. Nonzero runout

Right panels in Fig. 3 correspond to the milling model with runout parameters $\rho_1 = 0.9$ and $\rho_2 = 1.1$. In this case, the first tooth cuts 10% smaller and the second tooth cuts 10% larger than the ideal reference tool with zero runout. As it can be seen, this small difference between the load on the teeth does not affect the stability boundaries (the difference is infinitesimal), but it affects the structure of the frequency diagram qualitatively, since the principal period is now equal to the SR period $T$. The SR frequency and its harmonics are shown by dashed lines in the frequency diagram. It can be seen that all the three types of instability occur. At point C, there is period 2 chatter: the chatter frequencies are equal to multiples plus half of the SR frequency. At point D, quasi-periodic chatter arises. At point E, period 1 chatter arises: the chatter frequencies are
equal to the multiples of the SR frequency. The period 2 frequencies for point C are denoted by diamonds, the quasi-periodic frequencies for point D are denoted by squares and the period 1 frequencies for point E are denoted by triangles. Black filled markers denote the frequencies caused by the runout. Empty marker shapes denote the frequencies that arise in the zero runout model, too. The corresponding locations of the characteristic multipliers are shown in the bottom of Fig. 3.

4.3. Comparison

It can be seen that runout does not significantly change the stability boundaries, but it qualitatively affects the chatter frequencies. The number of chatter frequencies is doubled for the nonzero runout model. The period 2 chatter of the zero runout model turns to period 1 chatter in the nonzero runout model since the principal period changes from \( \tau \) to \( T = N \tau = 2\tau \). At point C, a special case of quasi-periodic chatter can be observed: the quasi-periodic frequencies cross each other resulting period 2 chatter. This corresponds to a double characteristic multiplier at \(-1\).

In this case, a tool with \( N = 2 \) teeth was considered, however, similar coincidence of the chatter frequencies and the SR frequencies arise for any tool with even number of cutting teeth.

5. Experimental verification

Cutting tests were performed on a 5-axis linear motor Ingersol machining center with a Fischer 40,000 rpm, 40 kW spindle. A 12.75 mm diameter, 106 mm overhang, carbide end mill was used during all stability tests. The modal parameters of the compliant tool are

\[
M = \begin{pmatrix}
0.046 & 0 \\
0 & 0.046
\end{pmatrix} \text{kg}, \quad C = \begin{pmatrix}
43.2 & 0 \\
0 & 43.2
\end{pmatrix} \text{Ns/m},
\]

\[
K = \begin{pmatrix}
9.57 & 0 \\
0 & 9.57
\end{pmatrix} \times 10^5 \text{N/m}.
\]

Cutting coefficients in the tangential and normal directions were determined during separate cutting tests on a Kistler Model 9255B rigid dynamometer. The estimated cutting coefficient values for the aluminum 7050-T7451 material were \( K_t = 536 \text{ N/mm}^2 \) and \( K_n = 187 \text{ N/mm}^2 \). An aluminum 7050-T7451 block was down-milled at a 5% radial immersion and a feedrate of 0.127 mm/tooth. The spindle speed \( \Omega \) and depth of cut \( a_p \) were changed for each cutting test to determine the onset of unstable vibrations. The runout parameters were estimated based on the cutting forces acting on the different teeth (see Fig. 2): \( \rho_1 = 0.9 \), \( \rho_2 = 1.1 \). More details on the cutting tests can be found in [9].

The experimental stability charts and the theoretical predictions are shown in the top left panel of Fig. 4. Stable cutting tests are denoted by circles, unstable cuts are denoted by crosses, and limit cases are denoted by + markers. The theoretical stability boundaries are denoted by thick lines. As it can be seen, there are some quantitative disagreements between the predicted and the experimental stability charts. The theoretical stability boundaries are higher along the depth of cut axis than the experimental one. This phenomenon can be caused by the uncertainty of the technological parameters like the cutting coefficients. Still, the predicted and the experimental stability charts agree well qualitatively (i.e., with respect to the structure of the stability boundary along the spindle speed).

The bottom left panel of Fig. 4 shows the theoretically predicted frequency diagram computed from the critical characteristic multipliers. Multiples of the SR frequency are denoted by a circle for all the four points P, Q, R and S, since these are the harmonics of the forcing frequency that are always present in the system’s response. In the period 2 case (point Q), chatter frequencies are denoted by diamonds. In the quasi-periodic case (point R), chatter frequencies are denoted by squares. In the period 1 case (point S), chatter frequencies are denoted by triangles. Black filled markers denote the frequencies caused by the runout.

PSD plots corresponding to cutting tests at points P, Q, R and S are shown in the right panels of Fig. 4. The SR frequency and its harmonics are denoted by dotted lines. Again, circles, squares, diamonds and triangles denote the SR, the quasi-periodic, the period 2 and the period 1 frequencies, respectively, and black filled markers denote the frequencies due to the runout. The different peaks in the PSD diagram are in good agreement with the theoretically predicted chatter frequencies. Note that logarithmic scale is used on the vertical axis in order to show the peaks associated with higher harmonics.

For period 2 (point Q) and quasi-periodic (point R) cases, the presence of chatter can clearly be assessed based on frequency spectrum. For period 1 chatter (point S), the chatter frequencies coincide with the harmonics of the SR frequency, resulting in a spectrum that is qualitatively identical to the spectrum of stable machining (see point P). Therefore, for period 1 case, the presence of chatter cannot be detected based on the frequency spectra. In order to overcome this problem, the vibration signals and their Poincaré (or stroboscopic) sections are analyzed directly, since they contain more information about the process than the frequency spectra. The structure of the Poincaré sections clearly refers to the dynamic behavior of the system (see, e.g. [20,24]).

6. Poincaré sections

In order to show the difference between period 1 chatter (point S) and stable machining (point P), \( 1/\tau \) sampled time history of the tool motion is analyzed both numerically and
experimentally. Poincaré sections are obtained by plotting the stroboscopically sampled data in the $x-y$ plane. 

Fig. 5 shows the $1/\tau$ sampled time history and the corresponding Poincaré sections obtained by the numerical simulation of Eq. (1). The loss of contact between the tool and the workpiece was also incorporated into the simulation in order to show the arising large-amplitude vibrations. Note that the sampling period is equal to the TP period $\tau$ and not to the SR period $T$. Every second sampling is denoted by gray color. This way, grey color refers to the position of the tool when tooth (1) is in the cut, while black color refers to the position of the tool when tooth (2) is cutting. It can clearly be seen that in the stable case, the position of the tool is almost the same at each $1/\tau$ sampling. This small difference is caused by the different load acting on the different teeth, e.g., by the runout of the tool. In the period 1 case, this difference is more significant, one of the teeth (black) experiences essentially larger load than the other (grey). This is caused by the period 1 chatter. This difference is more obvious in the Poincaré sections. In the stable case, the grey and black points are very close to each other, they have about the same phase, while in the period 1 case, they are clearly separated, and the phase between them is about $180^\circ$.

Fig. 6 shows the $1/\tau$ sampled time history and the corresponding Poincaré sections that were recorded during the cutting tests. It can be seen that qualitative behavior of the system is the same as it was predicted in Fig. 5. Thus, the phase of the points in the Poincaré sections can be used to distinguish between period 1 and stable cases.

7. Conclusions

The detection of machine tool chatter was investigated for the case of cutter runout based on frequency spectra and vibration signals. Due to the runout, the system is periodic at the SR period $T$, therefore this period was used to assess different types of chatter phenomena instead of the TP period $\tau$ that is generally used in the literature. The analysis was performed according to the Floquet theory for periodic systems. A two fluted tool was investigated, and it was shown that the chatter that is referred to as period doubling in the literature is in fact a period 1 chatter for the case of runout, since the SR period is just double of the TP period.

The distinction between period 1 chatter and stable machining operations was assessed from the $1/\tau$ sampled periodic motion resulted in two points in the Poincaré section. For stable machining, these points have about the same phase, while in the period 1 case, they are clearly separated, and the phase between them is about $180^\circ$. The results were clearly confirmed by experiments.
In this paper, a tool with \( N = 2 \) teeth was considered, however, similar coincidence of the chatter frequencies and the SR frequencies arise for any tool with even number of teeth. For these cases, the Poincaré sections can be used to distinguish between stable and unstable cases similarly as it was done in the current study.

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