On Stability and Dynamics of Milling at Small Radial Immersion

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Abstract
Stability and dynamics of milling at small radial immersion are investigated. Stability charts are predicted by the Semi Discretization method. Two types of instability are predicted corresponding to quasiperiodic and periodic chatter. The quasiperiodic chatter lobes are open and distributed along the spindle speed axis only, while the periodic chatter lobes are closed curves distributed in the plane of spindle speed and depth of cut. Experiments confirm the stability predictions, revealing the two principal types of chatter, the bounded periodic chatter lobes, and some special chatter cases. The recorded tool deflections in these cutting regimes are studied. The experiments also show that the modal properties of a slender tool may depend on spindle speed.

Keywords:
End milling, Stability, Dynamics

1 INTRODUCTION
Milling operations with small radial immersion and long slender tools are often required in finish machining of parts with deep pockets and thin walls. Since such flexible tools and parts are very susceptible to chatter vibrations, accurate manufacturing can be assured only by judicious selection of cutting parameters. Prediction of chatter free cutting parameters has been intensively investigated over the last decades [1]. Recently it has been shown that stability properties of milling change significantly at small radial immersions where cutting becomes highly intermittent [2]. Further analytical, numerical and experimental investigations [3][4][5][6] have confirmed that in cases of very small immersion milling the true stability boundary differs significantly from the approximate one predicted by the Single Frequency Solution (SFS) method [7], which is widely used for prediction of chatter-free parameters in milling. The main difference is associated with an additional type of instability which occurs during highly intermittent cutting. This instability is called period doubling or flip bifurcation and causes periodic chatter vibrations. The other type of instability, so far considered as the only one in milling, is called Hopf bifurcation and causes quasiperiodic chatter. Since there is a set of stability lobes associated to the each type of instability, the stability boundary in small immersion milling is composed of two sets of lobes located at different spindle speeds.

This paper reports results of analytical and experimental investigations of milling stability at small radial immersion. Stability boundaries are predicted by the Semi Discretization (SD) method [8]. They are indeed composed of two sets of stability lobes, respectively corresponding to the quasi-periodic and periodic chatter. However, in contrast to the quasiperiodic chatter lobes that are open and distributed along the spindle speed axis, the periodic chatter lobes are shown to be closed curves distributed in the plane of spindle speed and depth of cut. The stability predictions are confirmed by experiments that reveal the two principal types of chatter and some special chatter cases, and also indicate boundedness of periodic chatter lobes. Tool deflections recorded during the observed motion types are studied in detail. The experiments also show that modal properties of a long and slender tool may depend on spindle speed.

2 MATHEMATICAL MODEL OF 2-DOF END MILLING
Consider a 2-dof milling operation sketched in Figure 1.

![Figure 1: Sketch of 2-dof end milling.](image)

A cutter with \( N \) equally spaced teeth rotates at a constant angular velocity \( \Omega \). The radial immersion angle of the \( j \)th tooth varies with time as \( \phi(t) = 2\pi(\Omega t + (j-1)/N) \). A compliant machine tool structure is excited by the cutting forces at the tool tip causing dynamic response of the structure governed by the following equation:

\[
M\ddot{X}(t) + C\dot{X}(t) + KX(t) = F(t)
\]

(1)

Here \( X \) and \( F \) denote the displacement and cutting force vectors, while \( M, C, \) and \( K \) denote the mass, damping and stiffness matrices. The cutting force components acting on the \( j \)th tooth are given by:

\[
F_{x,j} = g_j(t)\left[-F_{r,j}(t)\cos\phi_j(t) - F_{t,j}(t)\sin\phi_j(t)\right]
\]

\[
F_{y,j} = g_j(t)\left[F_{t,j}(t)\sin\phi_j(t) - F_{r,j}(t)\cos\phi_j(t)\right]
\]

(2)

where \( g_j(t) \) is a unit step function determining whether or not the \( j \)th tooth is cutting. The tangential and radial cutting force components, \( F_{t,j} \) and \( F_{r,j} \), are assumed proportional to
the chip load defined by the product of chip thickness \( h_j(t) \) and depth of cut \( a_p \) as:

\[
F_{ij}(t) = K a_p h_j(t), \quad F_{ij}(t) = k_i F_{ij}(t)
\]

where \( K \) and \( k_i \) respectively denote the specific tangential force coefficient and the force ratio. The chip thickness consists of a static part due to feed, \( f_s \sin \phi_j(t) \), and a dynamic part due to cutter displacement. The stability of cutting is influenced only by the dynamic part of chip thickness given by:

\[
h_j(t) = g_j(t) \left( \Delta x \sin \phi_j(t) + \Delta y \cos \phi_j(t) \right)
\]

where \( \Delta x \equiv x(t-T) \) and \( \Delta y \equiv y(t-y(t-T)) \) describe the surface regeneration, i.e., the difference between the tool positions at the present and previous tooth passes. \( T = 2 \pi / \Omega \) denotes the tooth passing period.

Summing the contributions of all cutting edges yields the total cutting force:

\[
\left[ \begin{array}{c} F_x(t) \\ F_y(t) \\ \end{array} \right] = a_p K [ A_{xx}(t) A_{xy}(t) \\
A_{yx}(t) A_{yy}(t) ] [ \Delta x(t,T) \Delta y(t,T) ]
\]

where \( A_j(t) \) denote the time periodic directional dynamic force coefficients (see [7][9] for details). The governing equation of motion of a milling cutter therefore reads:

\[
M \ddot{x}(t) + C x(t) + K x(t) = a_p K A(t)(x(t) - x(t-T))
\]

Time dependence of the directional coefficients \( A(t) \) complicates the linear stability analysis of Eq. (6). A possible solution to this problem, followed by the Multi Frequency Solution (MFS) and SFS methods, is to expand \( A(t) \) in a Fourier series and retain the terms necessary for the approximation. In the MFS method [6][9], several Fourier terms are retained, whereas in the SFS method [7] only the zeroth order term is kept. The latter approximation is very practical, as it allows a closed form expression of the stability boundary, but it loses accuracy as the radial immersion and the number of cutter teeth decrease, leading to highly intermittent cutting. Alternatively, stability of Eq. (6) can be studied by the recently proposed time domain methods, the Temporal Finite Element Analysis [3][10] or Semi Discretization [3][8] methods. The latter is used in this study and is briefly reviewed below.

3 SEMI DISCRETIZATION METHOD

The basic idea of the Semi Discretization method is to discretize the delay terms of the delay differential equation (DDE) while leaving the current time terms unchanged. This way, the DDE is approximated by a series of ordinary differential equations (ODEs) for which the solutions are known and can be given in closed form [8].

The governing equation of milling (Eq. (6)) is a delayed differential equation with the tooth passing period \( T \) as delay. Using \( Q(t) = a_p K A(t) \) to simplify the notation, Eq. (6) may be rewritten as:

\[
M \ddot{x}(t) + C x(t) + K x(t) = Q(t)(x(t) - x(t-T))
\]

Discretization is introduced using a time interval \( [t_i, t_i+1) \) with \( t_i = i T + \Delta t \). The delay time becomes \( T = (m + 0.5) \Delta t \) where \( m \) is an integer defining the coarseness of the discretization. The periodic coefficient \( Q(t) = Q(t+T) \) and the delayed state \( x(t-T) \) are approximated by:

\[
Q(t) = Q_j, \quad x(t-T) = 0.5(x_{j-m+1} + x_{j-m})
\]

The DDE in Eq. (7) is hereewith transformed into a series of autonomous second order ODEs with \( t_i \leq t < t_{i+1} \):

\[
M \ddot{x}(t) + C x(t) + (K + Q_j) x(t) = Q_j \left( x_{j-m+1} + x_{j-m} \right)
\]

which can be rewritten as systems of first order ODEs:

\[
u(t) = W_i u(t) + V_j (u_{j-m+1} + u_{j-m}) = W_i u(t) + w_j
\]

with \( u = [x, y, x, y] \). Given the initial condition \( u(t_0) = u_0 \), the solution of Eq. (10) is:

\[
u(t) = e^{W_i (t-t_0)} [u_0 + W_i^{-1} w_j] + W_i^{-1} w_j
\]

Substituting \( t = t_{i+1} \) and \( u(t_{i+1}) = u_{i+1} \) into this solution yields:

\[
u_{i+1} = e^{W_i W_j} u_i + (e^{W_i W_j} - I) W_i^{-1} V_j (u_{j-m+1} + u_{j-m})
\]

Eq. (12) can be rewritten as a map: \( v_{i+1} = Z_i v_i \), with the state vector \( v_i = [u_i, u_{i-1}, \ldots, u_{i-m}] \) and the coefficient matrix:

\[
Z_i = \begin{bmatrix}
P_i & 0 & 0 & \ldots & 0 & R_i & R_i \\
1 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 1 & 0 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & 1 & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & 1 & 0
\end{bmatrix}
\]

The transition matrix over the principal period \( T \) is approximated by coupling the solutions of \( m \) successive intervals as:

\[
\Phi = Z_{m-1} Z_{m-2} \cdots Z_1 Z_0
\]

Finally, stability of the investigated system is determined by the eigenvalues of the transition matrix \( \Phi \). The system is stable if all eigenvalues of \( \Phi \) are in modulus less than 1. Further details on the semi discretization procedure can be found in [8].

In the case of milling, two possible instabilities can be observed:

1. The critical eigenvalue of \( \Phi \) is complex and its modulus is greater than 1. This case corresponds to the Hopf bifurcation causing the quasiperiodic chatter.
2. The critical eigenvalue of \( \Phi \) is real and its value is smaller than –1. This case corresponds to the periodic doubling or flip bifurcation which causes the periodic chatter.

Two these instabilities are illustrated in Figure 2 by the eigenvalue trajectories in the complex plane accompanied by the stability boundary with the corresponding depth of cut and spindle speed values. In the case of Hopf bifurcation, a pair of complex conjugate eigenvalues penetrates the unit circle in the complex plane, whereas in the case of flip bifurcation, the unit circle is penetrated by one real and negative eigenvalue. More information on bifurcations in dynamical systems can be found in [11].

4 RESULTS

The cutting tests were conducted on a high speed milling center using a cylindrical end mill with a single cutting edge (\( N = 1 \)), \( D = 8 \) mm diameter, 45 degree helix angle, and \( L = 69 \) mm overhang (L/D=12). A relatively large overhang was used to assure a single dominant vibration mode of the tool, whereas a single edged cutter was used to avoid the disturbances due to tool runout. The purpose of these two
simplifications was to provide clearer demonstration and better understanding of the dynamic properties of milling. The cutter was mounted in a HSK40E shrink fit holder. The workpiece was a square block made of AlMgSi0.5 aluminum alloy, for which the specific tangential force coefficient and the force ratio were determined mechanistically [12][13]: $K_t=644$ MPa and $K_s=0.37$. Minimal amount of coolant was used.

Tool deflections during cutting were measured in $x$ and $y$ directions simultaneously by a couple of laser optical displacement sensors mounted on the spindle housing. The sampling rate of the sensors was 10 kHz. A photo diode attached to the spindle housing provided a signal synchronized with the spindle rotation which was used in off-line stroboscope resampling of the deflection records.

The FRFs of the machine-tool structure at the tool tip (tip-tip FRFs) was determined mechanistically [12][13]. At $n=644$ MPa and $K_s=0.37$, minimal amount of coolant was used. Tool deflections during cutting were measured in $x$ and $y$ directions simultaneously by a couple of laser optical displacement sensors mounted on the spindle housing. The sampling rate of the sensors was 10 kHz. A photo diode attached to the spindle housing provided a signal synchronized with the spindle rotation which was used in off-line stroboscope resampling of the deflection records.

4.1 Identification of modal parameters
The frequency response function (FRF) matrix of the machine-tool structure at the tool tip (tip-tip FRFs) was determined by an impact test procedure. Due to the flexible tool of a relatively small diameter, impulse-like excitation at the tool tip was difficult to achieve. Regular excitation could only be assured by hitting the tool at the shaft below the tool holder. The tip-tip FRF could therefore not be measured directly; instead it had to be calculated from the FRFs with excitation at the tool shaft. To facilitate such a calculation, the tool response was measured simultaneously at the excitation point (15 mm below the tool holder) and at the tool tip, yielding shaft-shaft and shaft-tip FRFs. In order to check for the presence of mode coupling, the tool response was measured simultaneously in $x$ and $y$ directions at both locations. For this purpose, two pairs of low mass accelerometers (0.7 g each) were attached to the tool at $a_e$. Only these additional sets of lobes correspond mainly to the flip bifurcation instability which causes quasiperiodic chatter vibrations. As $a_e$ is increased, an additional set of lobes appears with stability maxima at spindle speeds equal approximately twice the odd integer fractions of the dominant eigenfrequency $f_e$.

4.2 Predicted stability charts
Stability charts predicted by the SD method for up milling and a series of radial immersions $a_e$ are shown in Figure 3. For $a_e=0.5D$, a single set of lobes is observed in the chart, with stability maxima located at spindle speeds equal to the integer fractions of the dominant eigenfrequency $f_e$, $n=f_e/(khN)=60$ rpm. For $k=2$, 3, and 4 and $N=1$, the stability maxima are located at approximately 21.7, 14.4 and 10.8 krpm. This set of lobes corresponds to the Hopf bifurcation instability which causes quasiperiodic chatter vibrations. As $a_e$ is increased, an additional set of lobes appears with stability maxima at spindle speeds equal approximately twice the odd integer fractions of the dominant eigenfrequency $f_e$, $n=2f_e/(2k+1)N=60$ rpm. Found at approximately 28.9, 17.3 and 12.4 krpm, these additional lobes correspond mainly to the flip bifurcation instability which causes periodic chatter.

4.3 Experimental stability charts
The predicted stability boundaries were verified experimentally by cutting tests conducted at a series of spindle speeds $n$ and depths of cut $a_e$. During each cut, a 100 mm long straight path was machined. Stability of cutting was assessed based on the recorded tool deflections, sound emitted during cutting, and roughness of the machined sur-

![Figure 3: Predicted stability charts for up milling and a series of radial immersions $a_e$.](image)

Table 1: Diagonal elements of the system matrices for the tip-tip FRFs.

<table>
<thead>
<tr>
<th>Mass [g]</th>
<th>Damping [kg/s]</th>
<th>Stiffness [kN/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>xx</td>
<td>20.1</td>
<td>1.56</td>
</tr>
<tr>
<td>yy</td>
<td>19.9</td>
<td>1.60</td>
</tr>
</tbody>
</table>
The experimental and predicted stability charts for up-milling with $a_e=0.5D$, 0.25D, and 0.05D are compared in Figure 5. The results for down milling and other radial immersions are similar (not shown).

Figure 4: Stability boundary for up milling at $a_e=0.05D$ with flip bifurcation lobes outlined.

Figure 5: Predicted and experimental stability charts for up milling at various radial immersions $a_e$.

For $a_e=0.5D$, the locations and amplitudes of experimental stability maxima agree well with predictions, while the amplitudes of experimental stability minima increase systematically with increasing spindle speed. A flip lobe is found at $n=29$ krpm, as predicted. For $a_e=0.25D$, locations of the experimental stability maxima still match approximately the predicted ones, while their amplitudes are significantly lower than predicted. An increasing trend in experimental stability minima is also observed. The flip lobes are found at $n=28$, 18, and 12 krpm, all slightly below the predicted locations. The situation for $a_e=0.05D$ is similar. Note, however, that the flip lobes at 12 and 17 krpm appear to be bounded from above by the Hopf lobe, while the lobe at 28 krpm narrows above, similarly to a lens-like shape. This indicates that the flip lobes might indeed be closed lens-like curves as predicted.

The observed shift of the lobes toward spindle speeds lower than predicted could be explained by variation of the dominant eigenfrequency of the tool with spindle speed. To verify this assumption, the tool response to the impulse excitation was measured by the laser optical sensors at a series of spindle speeds. Due to the limitations of the experimental setup, these FRFs could not be used for stability prediction, but they do illustrate the influence of spindle speed on modal properties of the tool. The experiments show that the frequency and compliance of the dominant vibration mode both decrease as spindle speed is increased. Figure 6 depicts the variation of the frequency $f_t$ for the spindle speed range considered. At its minimum at 23 krpm, $f_t=684$ Hz is approximately 5% lower than $f_t=722$ Hz at 0 krpm. If 684 Hz is used to calculate the location of the second flip lobe, for example, $n=27.4$ krpm is obtained instead of 28.9 krpm, which agrees very well with the lobe’s location in the experimental stability chart for $a_e=0.25D$ and 0.05D (Figure 5). Decrease of the mode compliance with increasing spindle speed (not shown) could also cause the systematic increase of the experimental stability minima which was observed for all $a_e$.

Figure 6: Frequency of the dominant vibration mode of the rotating tool versus spindle speed.

4.4 Tool paths in $(x, y)$ plane

Typical examples of tool motion in $(x, y)$ plane for the periodic chatter free, quasiperiodic chatter and periodic chatter regimes are shown in Figures 7, 8 and 9 (panels (a)), together with the stroboscopically sampled deflection in the feed $(x)$ direction versus time (panels (b)) and the amplitude spectrum of $(x$-deflection (panels (c)). Frequency of the stroboscope was set at the tooth passing frequency $f_s=17/T_n$, which is $f_s=0$ if $n=1$. Possible causes of the stroboscopic sampling yielded exactly one data point per tooth revolution. For the two periodic cases (Figures 7 and 9), the noise-free trajectories of the tool motion are shown superimposed on the recorded trajectories, providing a detailed picture of the tool path which would have been observed if there had been no noisy influences on the tool during cutting. The noise-free trajectories were obtained by a nonlinear filtering technique suitable for periodically
forced processes [15]. All examples shown in Figures 7, 8 and 9 are taken from the up milling cuts with \( a_p = 0.05D \).

In chatter free regime (Figure 7), the tool oscillates periodically with the tooth passing frequency \( f_p \) which means that the tool motion repeats itself after each tooth pass. The stroboscopically sampled data points form an ellipse (panel (a)) and their values remain approximately constant as cutting progresses (panel (b)). The amplitude spectrum of the deflection contains peaks only at the multiples of \( f_p \) (denoted by vertical lines in panels (c)).

Tool motion in periodic chatter regime (Figure 9) is also periodic, but with twice the tooth passing period \( 2T_p \) (or half the tooth passing frequency \( f_p/2 \)). This means that the tool motion repeats itself after two tooth passes. The stroboscopically sampled data points form two compact clouds which are visited alternately by the tool trajectory. The amplitude spectrum of the deflection shows peaks at the multiples of \( f_p/2 \). Comparison of all three amplitude spectra in Figures 7, 8 and 9 confirms that the tooth passing frequency is always present in the process, since it corresponds to the excited vibration of the tool, while the appearance of additional peaks in the spectra indicates chatter. This well known property of milling dynamics is often exploited for the purpose of chatter detection.

Finally, two special cases of chatter observed experimentally are presented: a combination of quasiperiodic and \( 2T \) periodic chatter (Figure 10), and a case of periodic chatter with period \( 3T \) (Figure 11). Periodic chatter cases with higher periods were also observed but less frequently.

In quasiperiodic chatter (Figure 8), the tool moves on a torus defined by the tooth passing frequency \( f_p \) and the dominant eigenfrequency \( f_t \) of the machine tool system. The stroboscopically sampled data points form an ellipse and their values oscillate in time. Peaks in the deflection spectrum are found at the two frequencies, \( f_p \) and \( f_t \), at their sums and differences, and multiples thereof.
and at least one real eigenvalue of the transition matrix (Eq. (14)) were in modulus larger than 1, i.e. outside the unit circle in the complex plane.

Periodic chatter cases with periods greater than 2T are in fact special cases of the quasiperiodic chatter. They were observed mainly at low radial immersions and always within the Hopf bifurcation lobe, where the quasiperiodic type of chatter was expected. In Figure 11, the periodic chatter example with 2T shown is the case shown here could therefore correspond to the quasi-periodic chatter case in which the pair of complex conjugate eigenvalues of the transition matrix penetrates the unit circle in the complex plane at angles ±2πm/3 (see Figure 2). From the point of view of the dynamical systems theory [11], occurrence of such periodic chatter cases could also be attributed to the fact that at small radial immersions the periodic motion synchronized with the periodic forcing attract the trajectory more than the quasiperiodic motion which is unsynchronized with the forcing.

5 CONCLUSIONS

Results of analytical and experimental investigations of milling stability and dynamics were presented. The investigations were focused on milling at small radial immersion using a long and slender tool. Stability boundaries predicted by the Semi Discretization method were composed of two sets of lobes corresponding to the Hopf and flip bifurcation instabilities which respectively cause quasiperiodic and periodic chatter. The Hopf lobes are open and distributed along the spindle speed axis only, while the flip lobes are lens-like closed curves distributed in the plane spanned by the spindle speed and axial depth of cut. Although much smaller than the Hopf lobes, the flip lobes often reach to lower cutting depths than the Hopf lobes and may be located partly or even entirely within the stable domain corresponding to the Hopf lobes.

Experiments confirmed the stability predictions. The Hopf and flip lobes were observed close to the predicted locations, and some of the flip lobes were found to be bounded by the Hopf lobes. In contrast to the good qualitative agreement between the predicted and experimental stability boundaries, significant quantitative discrepancies were observed at the stability maxima. Possible causes of these discrepancies could be the variation of the actual radial immersion due to the tool deflection, and the effect of edge forces, which were not considered in the force model. The experiments also showed that modal properties of a long and slender tool may vary with spindle speed.

Detailed study of the recorded tool deflections during chatter free, quasiperiodic and periodic chatter regimes confirmed the predicted properties of tool motion in these regimes and also revealed special cases of chatter rarely reported in the literature.

In summary, there indeed exist two types of instability in milling and both of them should be considered in prediction of chatter free parameters, particularly in cases of small radial immersion. However, further research of milling mechanics and dynamics is still required to improve the accuracy of predictions.

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7 REFERENCES