On the Nonsmooth Dynamics of Conventional Milling Processes

Zoltan Dombovari*, and Gabor Stepan*
Department of Applied Mechanics, Budapest University of Technology and Economics, Budapest, Hungary

Summary. The presented work shows an effective description of the phase space of conventional milling process, which is a time-periodic, nonlinear, delayed system due to the rotation of the milling tool, the cutting force characteristics and the appearing regenerative effect, respectively. It is shown that the tool can reach such large amplitude that it misses cuts and instead of a single delay its multiplied version operates. The vibration can be so violent that all teeth can leave the surface and the tool can actually fly over the surface bypassing any cuts. This effect results in the cutting force switching completely off and the dynamic system behaves as a finite dimensional system till one of the teeth bites back to the material again. This work shows parameter regions where these different kinds of effects can appear in the model of conventional milling processes.

Introduction

The dynamics of milling processes was investigated first by Tlusty [1] and Tobias [2] in the middle of the last century. Both recognized the effect of the regeneration of the past motion of the tool through the cut surface. Mathematically the system can be represented by delay differential equations (DDEs) which generate infinite dimensional phase space. In milling, this is subjected to time-periodic behaviour, which induces that the stationary solution is a time-periodic vibration. The asymptotic stability of this stationary cutting solution can be determined by the extended Floquet theory used on the perturbed variational system of the stationary cutting solution. In the literature there are many methods, which deal with the stability of time-periodic delayed variational systems. All these methods present the so-called stability lobe diagram. In it, along the stability border Hopf bifurcation or period doubling bifurcations of periodic orbits can appear in the nonlinear system. In an ideal situation, a perturbed system approaches the stationary solutions in the predicted stable domain, while in the unstable domain, the amplitude grows and reaches a threshold high amplitude vibration (chatter vibration). The linear stability charts were confirmed in many cases, although there are plenty of uncertainties involved still in predictions.

One of these uncertainties is caused by possible nonlinearities mostly originated from the nonlinear cutting force characteristics. In autonomous systems (like turning) it was shown that the nonlinearity induces subcritical Hopf bifurcation along the stability boundaries [3]. Similar results were shown in case of interrupted milling where period doubling bifurcations occur [4]. Furthermore, this behaviour was also demonstrated by experiments in case of conventional milling operation [5]. The subcritical sense means that there is an unstable orbit around the stable stationary solution, which might push the system to the high amplitude threshold chatter vibration depending on the level of perturbation.

In case of turning, it was shown that this stable threshold object exists due to the flyover effect, when the turning tool leaves the workpiece and jumps in again after a while. In [6] it was shown the unstable limit cycle can be extracted using centre manifold reduction and a so-called bistable region (unsafe zone) can be formulated below the linear stability boundaries where the system is sensitive for external perturbations.

Conventional Milling Model Subjected to Flyover Effect

In this model the structure of the milling machine (Figure $1a$) is characterized by linear modal analysis and it can be described by the following form assuming proportional damping

$$\ddot{\mathbf{q}}(t) + [2\xi_k\omega_{\mathrm{n},k}]\dot{\mathbf{q}}(t) + [\omega_{\mathrm{n},k}^2]\mathbf{q}(t) = \mathbf{U}^\mathsf{T}\mathbf{F}(t, \mathbf{x}(t), \mathbf{x}(t-\tau)), \tag{1}$$

where $\mathbf{x}(t) = \mathbf{U}\mathbf{q}(t)$ holds, if $\mathbf{x}$ and $\mathbf{q}$ are the Cartesian and modal coordinates. The mass normalized modal transformation matrix is $\mathbf{U} = [\mathbf{U}_1, \mathbf{U}_2, \dots, \mathbf{U}_m]$ for m selected modes. In (1) ξ_k and $\omega_{\mathrm{n},k}$ are the damping ratios and natural angular frequencies of the k^{th} mode ($k = 1, 2, \dots, m$). In simple cutting situation, due to the regeneration effect, the delay term $\mathbf{x}(t-\tau)$ appears with $\tau = 2\pi/\Omega/Z$ delay, where the constant angular velocity is Ω and the number of teeth is Z.

The cutting force $\mathbf{F}$ is originated from the specific empirical cutting force characteristics $\mathbf{f}(h) = \mathrm{col}(f_t(h), f_r(h), f_a(t))$ which depends mostly on the chip thickness h. In the industry this empirical cutting force characteristics are usually considered to be linear [7], but there are other more realistic nonlinear power-like and polynomial-like considerations, too. In order to ease the mathematical description we assume that the tool has cylindrical envelope geometry and it is straight fluted, that is, the helix angle is $\eta = 0$ and the lead angle is $\kappa = 90\,\mathrm{deg}$. Then the resultant cutting force can be expressed with the transformation matrix $\mathbf{T}$ [8] as

$$\mathbf{F}(t, \mathbf{x}(t), \mathbf{x}(t-\tau)) = -\sum_{i=1}^{Z} g_i(t)\mathbf{T}(\varphi_i(t))\mathbf{f}(g_i(t)h_i(t)), \tag{2}$$

where the angular position of the i^{th} tooth is $\varphi_i(t) = \Omega t + 2\pi(i-1)/Z$. We assume planar motion, that is, $\mathbf{x}(t) = \mathrm{col}(x_1(t), x_2(t))$ and small only x_1 directional feed motion characterized by the feed per tooth f_Z. In this case the

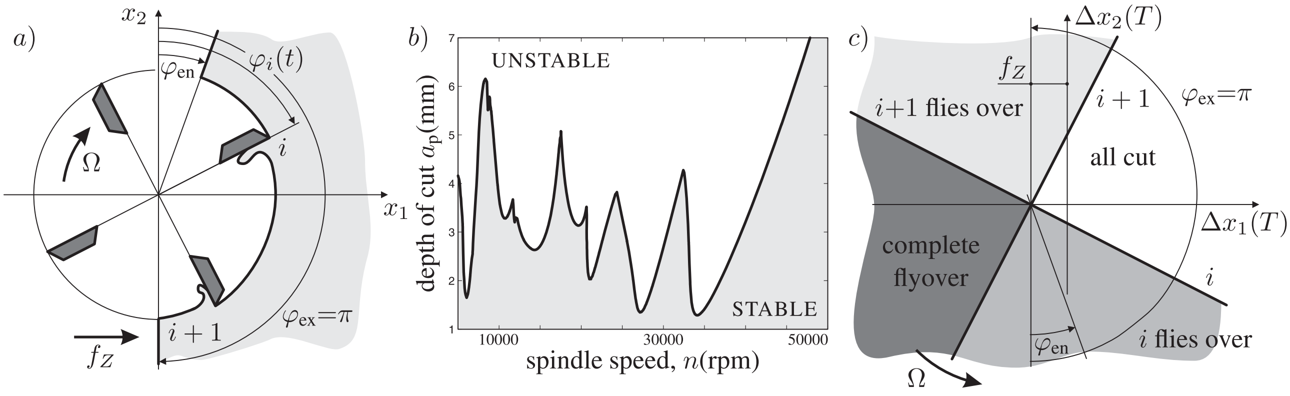

Figure 1: $a)$ sketch of milling process, $b)$ linear stability diagram, $c)$ flyover map of milling process.

originally trochoid path of the edges can be approximated by circles. Since $h_i(t)$ can be negative, real momentary chip thickness $g_i(t)h_i(t)$ is used in (2), which is characterized by the screen function $g_i(t) = \{1$, if $\varphi_{\text{en}} \leq \varphi_i(t) \bmod 2\pi \leq \varphi_{\text{ex}} \wedge h_i(t) > 0; 0$, otherwise$\}$, where φ_{en} and φ_{ex} are the enter and exit angles, respectively (Figure 1a).
If a tooth leaves the workpiece, the surface remains uncut which can be traced by the surface function, which is defined the following way

$$\boldsymbol{\chi}(t) = \text{col}(\chi_1(t), \chi_2(t)) = \begin{cases} \mathbf{x}(t), & \text{cut,} \\ \text{col}(\chi_1(t-\tau) - f_Z, \chi_2(t-\tau)), & \text{flyover.} \end{cases} \tag{3}$$

Accordingly, the theoretical momentary chip thickness cut by the i^{th} tooth is $h_i(t) := h_i(t, \mathbf{x}(t), \dots, \mathbf{x}(t - n_k\tau), \dots) = (f_Z + x_1(t) - \chi_1(t-\tau)) \sin\varphi_i(t) + (x_2(t) - \chi_2(t-\tau)) \cos\varphi_i(t)$, which means through $\boldsymbol{\chi}(t-\tau)$ many integer n_k ($k = 1, 2, \dots$) multiplications of delay τ appear in the system, that is, in (1) the resultant cutting force is $\mathbf{F}(t, \mathbf{x}(t), \dots, \mathbf{x}(t - n_k\tau), \dots)$. Different situations can appear during this violent chatter vibration. On the one hand there can be a tooth which leaves the surface but other teeth can remain in cut, which means the regenerative sense of the system is not violated. On the other hand vibration can reach that point where all teeth leave the surface, the resultant cutting force $\mathbf{F}$ switches off and the tool behaves as a simple high DOF damped vibratory system with an equilibrium at $\overline{\mathbf{q}} = \mathbf{0}$ till one of teeth hit the surface again.
This situation can be traced conveniently in a special transformation of the phase space using stroboscopic mapping by $T = \tau$ in this conventional milling case. The projection is made in the coordinate system of $(\Delta x_1, \Delta x_2)$, where $\Delta x_1(t) = x_1(t) - \chi_1(t-\tau)$ and $\Delta x_2(t) = x_2(t) - \chi_2(t-\tau))$. In this consideration for each teeth one can formulate the following conditions for cutting, if

$$\Delta y \lesseqgtr -\tan\varphi_i(t)(f_Z + \Delta x), \text{ if } \cos\varphi_i(t) \lesseqgtr 0, \quad i = 1, 2, \dots, Z. \tag{4}$$

These Z pieces of conditions, combined with φ_{en} and φ_{ex} angles, formulate domains in $(\Delta x_1(T), \Delta x_2(T))$ plane where one can follow which teeth are in cut and when the tool is flying over the surface of the material. In Figure 1c the 'flyover map' of $Z = 4$ fluted conventional milling is depicted with $3/4$ radial immersion.

Conclusions

The phase space structure of nonlinear, nonsmooth milling was explored in order to identify the presence of unstable limit cycles around stable stationary milling, which helps to determine unsafe zones close to the stability limits.
Acknowledgment: This work was supported by the Hungarian Scientic Research Foundation OTKA Grant no. K83890.

References

[1] Tlusty, J., and Spacek, L., 1954. *Self-excited vibrations on machine tools*. Nakl. CSAV, Prague. in Czech.

[2] Tobias, S., 1965. *Machine-tool Vibration*. Blackie, Glasgow.

[3] Kalmár-Nagy, T., Stepan, G., and Moon, F., 2001. "Subcritical Hopf bifurcation in the delay equation model for machine tool vibrations". *Nonlinear Dynamics*, ***26***(2), pp. 121–142.

[4] Szalai, R., Stépán, G., and Hogan, S., 2004. "Global dynamics of low immersion high-speed milling". *Chaos*, ***14***(4), pp. 1069–1077.

[5] Stepan, G., Dombovari, Z., and Muñoa, J., 2011. "Identification of cutting force characteristics based on chatter experiments". *CIRP Annals-Manufacturing Technology*, ***1***, pp. 113–116.

[6] Dombóvári, Z., Wilson, R., and Stépán, G., 2008. "Estimates of the bistable region in metal cutting". *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science*, ***464***(2100), pp. 3255–3271.

[7] Altintas, Y., 2000. *Manufacturing Automation: Metal Cutting Mechanics, Machine Tool Vibrations, and CNC Design*. Cambridge University Press, Cambridge.

[8] Dombovari, Z., Altintas, Y., and Stepan, G., 2010. "The effect of serration on mechanics and stability of milling cutters". *International Journal of Machine Tools and Manufacture*, ***50***(6), pp. 511 – 520.