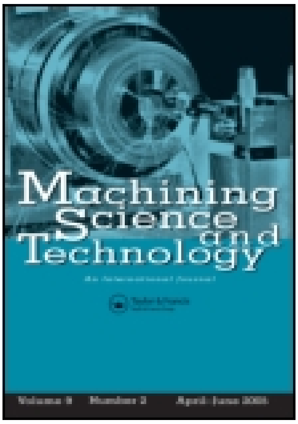

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Jokin Munoa [a], Zoltan Dombovari [b], Iker Mancisidor [a], Yiqing Yang [c] & Mikel Zatarain [a]

[a] Ideko-IK4 Technological Center, Elgoibar, Spain

[b] Department of Applied Mechanics, Budapest University of Technology and Economics, Budapest, Hungary

[c] School of Mechanical Engineering and Automation, Beihang University, Beijing, PR China

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Jokin Munoa[1], Zoltan Dombovari[2], Iker Mancisidor[1], Yiqing Yang[3], and Mikel Zatarain[1]
[1]*Ideko-IK4 Technological Center, Elgoibar, Spain*
[2]*Department of Applied Mechanics, Budapest University of Technology and Economics, Budapest, Hungary*
[3]*School of Mechanical Engineering and Automation, Beihang University, Beijing, PR China*

□ *The productivity of many industrial cutting processes is limited by high amplitude chatter vibrations. An optimization technique based on the use of the stability lobes helps to increase the productivity of these processes, improving the life of machine elements and reducing the tool wear as well. The best-known lobes correspond to Hopf bifurcations. However, in case of interrupted cutting, additional lobes appear due to period doubling or flip bifurcation. When the system has more than one dominant vibration mode, important variations can appear in stability due to interaction between modes. The basic mathematics for the appearance of these new lobes are shown in this article. The frequency domain study shows that lobes related to flip bifurcation are a special case of the interaction between modes. The results of these interactions are verified by comparison with semi-discretization method and time domain simulations, respectively.*

Keywords chatter, milling, stability

INTRODUCTION

In metal cutting, self-excited vibrations can limit productivity and put process safety at risk, giving rise to poor surface quality or even machine tool component failure. In industry, the use of stability diagrams has provided a practical way to select the optimum conditions. The regenerative effect was defined as the main cause of machine-tool chatter by Tobias (Tobias and Fishwick, 1958) and Tlusty (Tlusty and Polacek, 1963) almost simultaneously. Later, Merritt (1965) presented the problem as a feedback loop, clarifying the problem from an engineering point of view.

Address correspondence to Jokin Munoa, Ideko-IK4 Technological Center, Arriaga Kalea, 2, 20870, Elgoibar, Basque Country, Spain. E-mail: jmunoa@ideko.es

In milling processes, the governing equation is a time periodic piecewise smooth delay differential equation (DDE) (Hale, 1977; Stepan, 1989). Stability analysis is therefore much more complex than those representing continuous regenerative cutting processes. Nevertheless, significant progress has been made in the study of milling stability during the last decades.

In 1995, Altintas and Budak presented a semi-analytical method for chatter analysis in frequency domain. Later, Budak and Altintas (1998) worked out this solution by considering several terms of in the Fourier development of the directional matrix.

The single frequency approach has been shown to be very precise, but in case of low immersion milling, the existence of additional stability lobes related to period doubling or flip bifurcation was found. Davies et al. (2002) used a discrete map to model highly interrupted milling processes, where the engagement is infinitesimal and the cutting process is modelled as an impact. Insperger and Stepan (2000) used an approximation method called Fargue-method. Later, they developed the semi-discretization (SD) technique (Insperger and Stepan, 2002), and Bayly et al. (2002) developed the temporal finite elements method, obtaining similar results. Merdol and Altintas (2004) showed that the multi-frequency (MF) method is also able to represent accurately the period doubling instability phenomenon. At the same time, Corpus and Endres (2004) studied lobes related to flip bifurcations using an analytical approach.

Several authors have shown that some lens-shaped instability regions appear for period doubling zones of third and higher order in interrupted turning (Szalai and Stepan, 2006). By using the multi-frequency approach, Zatarain et al. (2006) showed that the tool helix produces the transformation of the added lobes (period doubling) into instability islands. Insperger et al. (2006) arrived to the same conclusion by using the semi-discretization method. Later, Patel et al. (2008) obtained the same results by means of temporal finite element method.

In industrial applications, such as steel face milling, it is common to find situations with more than one dominant mode at different frequencies. In this work, the interactions between different modes are considered and their effect on the stability chart is studied using the multi-frequency (MF) approach. Moreover, semi-discretization (SD) (Insperger and Stepan, 2011) method and time domain milling simulations (TDS) are used to validate the results. The objective of this article is to analyze the zero-order approximation (ZOA) performance for the general case, considering more than one dominant mode. New inaccuracies of the ZOA have been found.

FREQUENCY DOMAIN MILLING STABILITY MODEL IN MODAL COORDINATES

The characteristic equation for milling stability analysis in frequency domain was developed by Budak and Altintas (1998). Their approach was based on Cartesian displacements between the tool and the workpiece. In order to describe the regenerative effect, the dynamic cutting force is considered, and the relation between the force and the vibrations is developed. For face milling operations, the dynamic cutting force can be given in the following form (Altintas, 2003; Munoa et al., 2005):

$$\{F(t)\} = \left(\frac{K_t b}{\sin \kappa}\right)[A(t)]\{\Delta r(t)\} \tag{1}$$

where

$$\{\Delta r(t)\} = \{r_{\mathrm{t}}(t) - r_{\mathrm{t}}(t-\tau)\} - \{r_{\mathrm{w}}(t) - r_{\mathrm{w}}(t-\tau)\} \tag{2}$$

Here, $\Delta r(t)$ is the relative regenerative vibration between the tool (t) and the workpiece (w), b is the depth of cut (see Figure 1), K_t is the tangential cutting coefficient, τ is the tooth passing period, κ is the lead angle and $[A(t)]$ is the Cartesian directional factor matrix. The directional factor matrix concentrates the projection of the cutting force onto the mode direction and the projection of the vibration onto the chip thickness. The Cartesian approach leads to the use of the corresponding directional coefficient matrix $[A(t)]$.

The modal approach is more convenient for the present study and, for this purpose, the force expression has to be transformed into the modal space. Therefore, the relative normalized modal transformation matrix $[Q]$, the modal displacement $\{\eta\}$ and the modal force $\{P\}$ are introduced as

$$\{\Delta r(t)\} = [Q]\{\Delta\eta(t)\} \tag{3}$$

$$\{P(t)\} = [Q]^{\mathrm{T}}\{F(t)\} \tag{4}$$

where $\{\Delta\eta(t)\} = \{\eta(t) - \eta(t-\tau)\}, [Q] = [Q_{\mathrm{t}}] - [Q_{\mathrm{w}}], [Q_{\mathrm{t}}]$ *and* $[Q_{\mathrm{w}}]$ are the local modal transformation matrices of the tool and the workpiece. Hence, the dynamic milling force can be expressed in modal coordinates defining the modal directional factor matrix $[B(t)]$:

$$\{P(t)\} = -\left(\frac{K_t b}{\sin \kappa}\right)[B(t)]\{\Delta\eta(t)\} \tag{5}$$

where

$$[B(t)] = -[Q]^{\mathrm{T}}[A(t)][Q] \tag{6}$$

In milling, the dynamic cutting force is periodic at tooth passing period τ. In case of unstable stationary cutting, the vibration has a frequency (ω_c) close to one of the dominant natural frequencies and some modulations $\omega_{c,k}=\omega_c+k\Omega$ related to the tooth passing frequency $\Omega = 2\pi/\tau$. The frequency of these modulations depends on the number of flutes (Z) and the spindle speed (n). The next formulation can be considered for the modal milling force and vibration:

$$\{P(t)\} = \sum_{k=-\infty}^{\infty} \{P_k\} e^{\mathrm{j}(\omega_c+k\Omega)t} \tag{7}$$

and

$$\{\eta(t)\} = \sum_{r=-\infty}^{\infty} \{\eta_k\} e^{\mathrm{j}(\omega_c+r\Omega)t} \tag{8}$$

Considering this frequency pattern and operating, the regenerative term can be rewritten as

$$\{\Delta\eta\} = (1 - e^{-\mathrm{j}\omega_c\tau})\{\eta\}. \tag{9}$$

The modal directional matrix is also time periodic and, consequently, a discrete Fourier development is possible, thus

$$[B(t)] = \sum_{l=-\infty}^{\infty} [B_l]\, e^{\mathrm{j}\, l\,\Omega\, t} = \frac{Z}{2\pi}\left(\sum_{l=-\infty}^{\infty} [\beta_l]\, e^{\mathrm{j}\, l\,\Omega\, t} = \right) \tag{10}$$

Taking into account all the different developments in Equations (5) and operating with the different harmonics as a product of Equations (7), (8), and (10), the next expression is obtained

$$\sum_{k=-\infty}^{\infty} \{P_k\}\, e^{\mathrm{j}(\omega_c+k\,\Omega)t} = -\left(\frac{K_t\, b\, Z}{2\,\pi\ \sin\kappa}\right)(1 - e^{-\mathrm{j}\,\omega_c\,\tau}) \sum_{k=-\infty}^{\infty}\left(\sum_{r=-\infty}^{\infty} [\beta_{k-r}]\{\eta_r\}\right) e^{\mathrm{j}(\omega_c+k\,\Omega)t}. \tag{11}$$

The dynamic modal forces and displacements can be related using the dynamic properties of the mechanical structure. Therefore, considering the relative frequency response function (FRF) of the system [Φ] between tool (t) and workpiece (w), the next expression can be written for each harmonic component:

$$\{\eta_k\} = [\Phi(\omega_c + k\Omega)]\{P_k\} \tag{12}$$

where

$$[\Phi(\omega)] = \operatorname*{diag}_{i=1}^{N}\left(\frac{1}{m_i[(\omega_{\mathrm{n},i}^2 - \omega^2) - 2\mathrm{j}\xi_i\omega_{\mathrm{n},i}\omega]}\right) \tag{13}$$

Note that, $\omega_{n,i}$, ξ_i, and m_i, are, respectively, the natural frequency, the damping ratio, and the reflected modal mass of the i^{th} mode associated to the tool and the workpiece. Following Budak and Altintas's (1998) development, it is possible to obtain a closed loop formulation. The main equation relates different harmonics of the modal displacement η_k taking into account the directional factor matrices (10) for each harmonic and FRF matrices (12) evaluated at different harmonics. Therefore;

$$\{\eta_k\} = -\left(\frac{K_t bZ}{2\pi \sin\kappa}\right)(1 - e^{-\mathrm{j}\omega_c\tau}) \sum_{r=-\infty}^{\infty} [\Phi(\omega_c + k\Omega)][B_{k-r}]\{\eta_r\} \tag{14}$$

Finally, the stability problem results in an infinite dimensional matricial expression, that is,

$$\begin{Bmatrix} \vdots \\ \{\eta_{-h}\} \\ \vdots \\ \{\eta_0\} \\ \vdots \\ \{\eta_h\} \\ \vdots \end{Bmatrix} = -\left(\frac{K_t\, b\, Z}{2\pi\ \sin\kappa}\right)(1 - e^{-\mathrm{j}\omega_c\tau}) \begin{bmatrix} \ddots & \vdots & & \vdots & & \vdots & \dots \\ \cdots & [\Phi_{-h}] & \cdots & [0] & \cdots & [0] & \cdots \\ & \vdots & \ddots & \vdots & \ddots & & \\ \cdots & [0] & \cdots & [\Phi_0] & \cdots & [0] & \cdots \\ & \vdots & \ddots & \vdots & \ddots & \vdots & \\ \cdots & [0] & \cdots & [0] & \cdots & [\Phi_h] & \cdots \\ \dots & \vdots & & \vdots & & \vdots & \ddots \end{bmatrix}$$

$$\begin{bmatrix} \ddots & \vdots & & \vdots & & \vdots & \dots \\ \cdots & [\beta_0] & \cdots & [\beta_{-h}] & \cdots & [\beta_{-2h}] & \cdots \\ & \vdots & \ddots & \vdots & \ddots & & \\ \cdots & [\beta_h] & \cdots & [\beta_0] & \cdots & [\beta_{-h}] & \cdots \\ & \vdots & \ddots & \vdots & \ddots & \vdots & \\ \cdots & [\beta_{2h}] & \cdots & [\beta_h] & \cdots & [\beta_0] & \cdots \\ \dots & \vdots & & \vdots & & \vdots & \ddots \end{bmatrix} \begin{Bmatrix} \vdots \\ \{\eta_{-h}\} \\ \vdots \\ \{\eta_0\} \\ \vdots \\ \{\eta_h\} \\ \vdots \end{Bmatrix} \tag{15}$$

where $\{\eta_k\}$ are the amplitudes of vibration of all the considered modes for the k^{th} modulated frequency, and $[\Phi_k] = [\Phi(\omega_c + k\Omega)]$ and $[B_k]$ the FRF matrix and the modal directional factor matrix evaluated at the k^{th} modulated chatter frequency.

In a theoretical basis, the size of the matrices is infinite, but in practice the FRF takes very small values for frequencies far from the natural frequencies. Therefore, the system can be truncated without noticeable loss of accuracy.

The solution of this equation results in an eigenvalue problem where the obtained eigenvalues are related to the spindle speeds Ω/Z and depth of cuts b. Different numerical methods have been proposed to obtain the multi-frequency solution (Budak and Altintas, 1998; Merdol and Altintas, 2004). If only the zeroth-order term is considered, a fast semi-analytical solution is possible (Altintas and Budak, 1995).

For high tooth passing frequencies Ω, even a low matrix size in (15) produces vibration frequencies $\omega_c - \Omega$ with negative values. It must be noted that any vibration at frequency ω can be considered as the sum of two complex vibrators, at frequency ω and at its complex conjugate at $-\omega$. Therefore, when a spectrum contains some energy at frequency ω, the energy is shared among frequencies at ω and $-\omega$.

A formulation based on Cartesian displacements can also be used (Budak and Altintas, 1998), which may be useful because FRFs are often measured in Cartesian coordinates. Practically, when the number of considered modes is bigger than three, the modal approach gives rise to bigger matrix dimensions and larger calculation times. Next, the inaccuracies of the ZOA are analyzed, considering basically the MF analysis and comparing the results with SD and time domain milling simulations.

STABILITY LOBES WITH SEVERAL DOMINANT MODES

In general, a system with multiple modes refers more reliably to the otherwise high dimensional dynamics of a machine tool than a dynamical model with a single mode. In machine dynamics, a two-mode model is able to represent the complexity of a case with higher number of modes; whereas, single mode models do not capture the interactions between modes. Therefore, the results obtained for two modes offer a good view of dynamically complex machine tool structures by means of an affordable mathematical load. Most of the results can be extrapolated to the case of multiple modes. For this reason, a mechanical system with two dominant modes has been posed in this section. An example will be considered in order to describe the different inaccuracies of the ZOA (see Table 1) in a clearer way.

In a single mode system, the angles between mode and milling force, and between mode and chip thickness direction are really important. If

TABLE 1 Simulation Parameters

Tool				
Diameter, D(mm)	Number of flutes, Z	Helix angle, η(deg)	Lead angle, κ(deg)	
50	4	0	90	
Cutting conditions & coefficients				
K_t (N/mm^2)	K_r (N/mm^2)	Sense	Feed Direction (f)	
2000	600	Down Milling	(1,0,0)	
Dynamic Parameters				
i	$\omega_{n,i}$ (Hz)	ξ_i (%)	k_i (N/μm)	Orientation (v)
1	45	4	30	(1,0, 0)
2	60	4	30	(0,1,0)

systems with two modes are considered, there are more important parameters: the projection of both modes onto the chip thickness and cutting force, the difference between the frequencies of both modes and the differences between their damping ratios and modal stiffness. Therefore, it is complex to make a dimensionless study of the system. Two modes in perpendicular directions have been chosen with different natural frequencies avoiding the most special case when the modes are identical in their dynamic parameters. Remark that, in modal space, perpendicular modes are not a special case. With the chosen example the presence of the double period chatter and mode couplings is assured and, luckily, some stable islands exist.

Comparison between ZOA, Multi-frequency Solution, and Semi-discretization

Several simulations have been carried out using different down milling radial immersions (see Figure 2). Finally, the case of 25% radial engagement (RE) is studied. The dynamics and the cutting coefficients have been chosen according to standard conditions for steel face milling with universal milling machines (see Table 1). A straight-fluted tool with four teeth and a diameter of $D = 50$ mm has been selected to make the stability calculations. The lead angle of the tool is $\kappa = 90°$.

If the results between the ZOA and the multi-frequency methods are compared, it is clear that the results diverge in several zones of the lobe diagrams as the cutting force becomes more interrupted (see Figure 2(a)–(c)). If a linear model is considered, a slotting operation with four flutes produces a constant cutting force. In this case, there is no harmonics in the dynamic cutting force and in the modal directional matrix $[B]$; thus, the ZOA solution is exact. On the other hand, when the engagement is reduced or even when the slotting is performed using a tool with less than

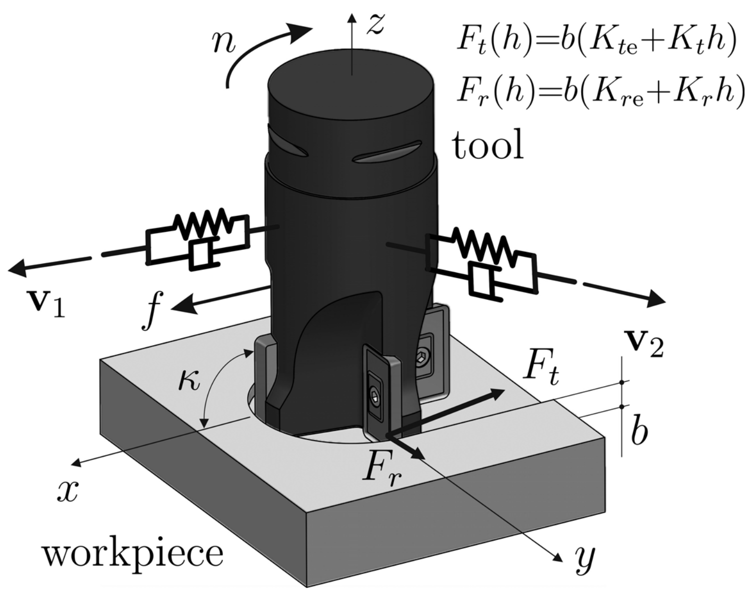


FIGURE 1 Schematic milling model.

four flutes, discrepancies among methods arise. Two different effects are creating these deviations in the case of a dynamic system with two or more modes: the period-doubling instability (flip bifurcation) and the mode interaction.

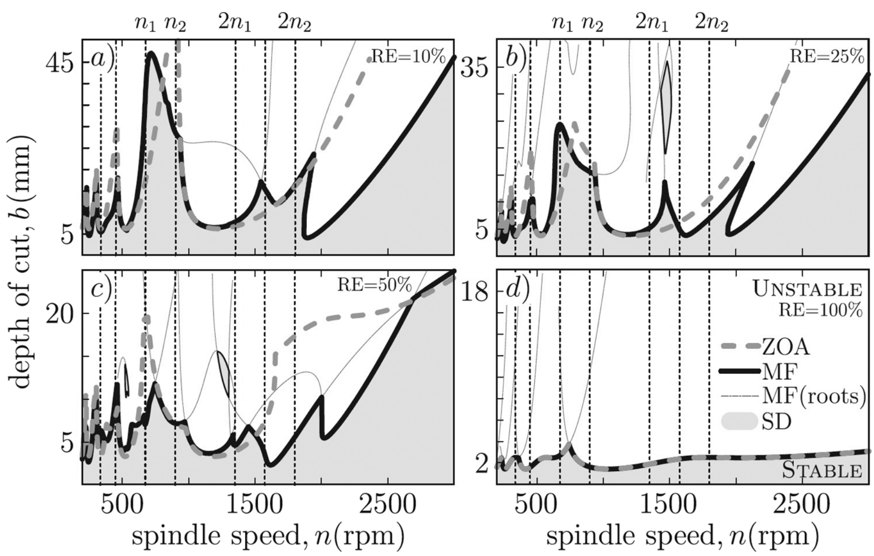


FIGURE 2 Stability charts in case of down milling processes with parameters taken from Table 1.

The semi-discretization method shows the presence of stable isolated zones for engagements of 25% and 50% (see Figure 2(b),(c)). The multi-frequency method also shows points limiting this stable region, but it is impossible to completely identify a stable zone due to the fact that the MF solution directly obtains the stability limits as a parametric function of the vibration frequency.

Period Doubling Instability

When angular immersion is small, an additional family of stability lobes appears, producing an increase of stability in a small area and a reduction in a larger zone. The main characteristic of these lobes can be recognized in its special spectral pattern in the frequency diagram (see Figure 3(a)), where the vibration (chatter) frequencies related to these additional lobes follow straight lines. Hence, chatter frequency has a direct relationship with cutting frequency.

$$\omega_c = (m/2)\Omega, \quad \text{where } m = 1, 3, 5, \ldots \tag{16}$$

If one point of this additional lobe is considered (see point C in Figure 3 and Figure 4) and studied in frequency domain, it can be seen that both the zeroth-order term and one of its harmonics (in this case −1 harmonic)

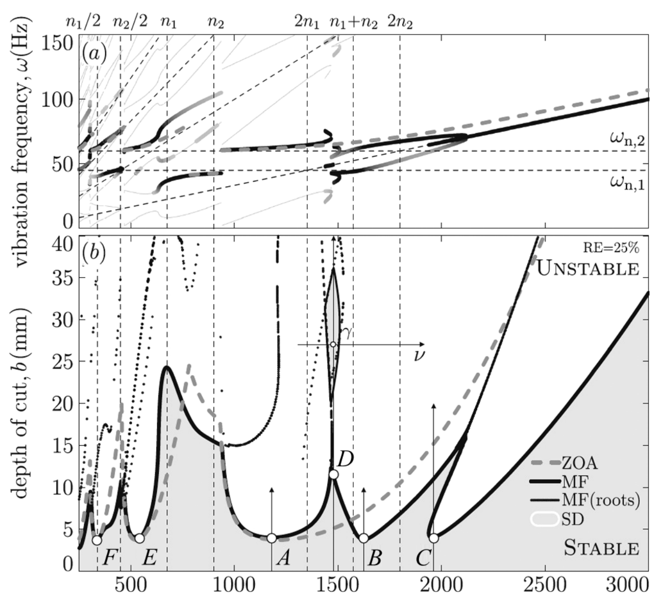


FIGURE 3 (a) The frequency plot of the critically stable solutions using grayscale showing the strengths of the harmonics (Dombovari et al., 2011). (b) The linear stability of milling process (cf. Table 1).

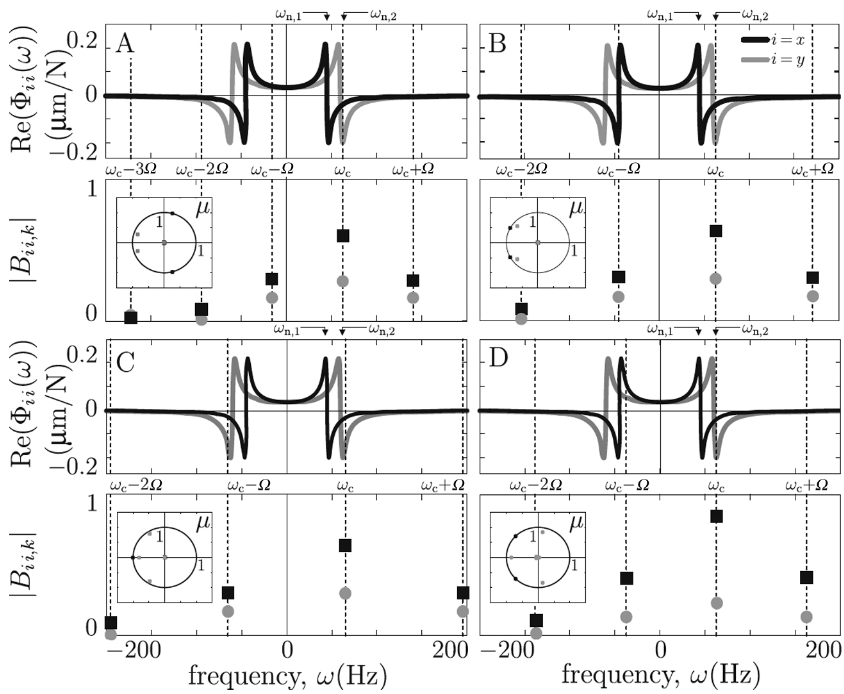


FIGURE 4 The panels show the effect of the mode interactions (see Figure 3). In each panel the frequency response functions (upper panel), the harmonics of the directional factors, and the unit circles with the critical eigenvalues are presented.

are exciting the same mode. Operating in double period chatter, the regenerative term of Equation (14) has the next form:

$$1 - e^{-j\omega_c \tau} = 1 - e^{-j\pi m} \tag{17}$$

Therefore, the regenerative term in case of double period chatter is a real number (Corpus and Endres, 2004).

$$\begin{cases} 1 - e^{-j\omega_c \tau} = 2, & m = \pm 1, \pm 3, \pm 5, \ldots, \\ 1 - e^{-j\omega_c \tau} = 0, & m = \pm 2, \pm 4, \pm 6, \ldots. \end{cases} \tag{18}$$

Only the odd harmonics can create this effect due to the fact that the even harmonics eliminate the regenerative term.

In the high-speed zone, the system can be truncated considering only the 0 and −1 harmonics, and the case of the double period chatter in point *C* can be studied accurately. For the main flip lobe, only harmonics 0 and −1 are relevant, and they both have equal amplitudes and different phases. Assuming that only one mode affects the stability in the period doubling instability in (15), the resulting equation is

$$\begin{Bmatrix} \eta_{-1} \\ \eta_0 \end{Bmatrix} = -\left(\frac{K_t\, b\, Z}{\pi\ \sin\kappa}\right)\begin{bmatrix} \Phi_{-1}\beta_0 & \Phi_{-1}\beta_{-1} \\ \Phi_0\,\beta_1 & \Phi_0\beta_0 \end{bmatrix}\begin{Bmatrix} \eta_{-1} \\ \eta_0 \end{Bmatrix} \tag{19}$$

where Φ_0 and Φ_{-1} are FRF at frequencies ω_c and $\omega_c - \Omega$, while β_0, β_1 and β_{-1} are the harmonics of the directional factor. It is remarkable that at period doubling stability boundaries the Φ_{-1} is the complex conjugate of Φ_0. In addition, β_1 and β_{-1} are also conjugates.

Considering all the special characteristics of the period doubling instability, it is straightforward to predict the spindle speed region zone affected by this phenomenon. The tooth passing frequency should be two times the natural frequency divided by an odd number, similarly as in (18)

$$\Omega = \frac{2\omega_n}{m}, \quad n = \frac{120\omega_n}{Z \cdot m}, \quad m = 1, 3, 5, \ldots \tag{20}$$

In our case study, this formula predicts the beginning of the double period chatter at 1800 r/min for the first harmonic and the second mode (60 Hz). The Flip bifurcation is not visible for the rest of the possible combinations (see $2n_2$ in Figure 3).

If the stability borders related to period doubling bifurcation are analyzed using the semi-discretization method (Insperger and Stepan, 2002), the unit circle shows that the instability is created when the dominant eigenvalue crosses the unit circle following the real axis (see point *C* in Figure 4). This means a possible periodic motion loses its stability by alternately in- and out-circling the stationary solution, that is, it flips in every revolution. The linear stability limits related to the period doubling (flip) bifurcation can have open or closed forms (Szalai and Stepan, 2006; Zatarain et al., 2006), although always independent from linear stability limits related to Hopf-bifurcation. The multi-frequency approach must be applied in frequency domain to capture these lobes.

Mode Interactions

When the system has two (or more) strong modes, another similar effect can happen. In some regions two modes start interfering and, consequently, this effect increases or reduces the linear stability depending on the sense of the interaction (see point *B* in Figures 3 and 4). The dynamic cutting force and the directional factor should have strong components to create this effect and, therefore, an interrupted milling process is necessary.

In the case of mode interaction, there is not a special distribution of the chatter frequencies or the stability boundaries. Hence, the borders can only be obtained using pure numerical methods. Anyway, considering that the chatter frequency is always very close to the natural frequencies, it is possible to predict the range of milling frequencies considering the physical meaning and different harmonics.

$$\left.\begin{aligned} \Omega &= \frac{\omega_{\mathrm{n},1} + \omega_{\mathrm{n},2}}{m}, \\ \Omega &= \frac{\left|\omega_{\mathrm{n},1} - \omega_{\mathrm{n},2}\right|}{m}, \end{aligned}\right. \quad m = 1, 2, 3, 4, 5, \ldots \tag{21}$$

Note that (21) can be generalized for more than two modes and the spindle speed can be calculated as $n = 60\,\Omega/Z$. The point B in Figure 3 represents this phenomenon when formula (21) shows that the first harmonic is creating interference around 1575 r/min, taking into account the sum of both modes (see $n_1 + n_2$ in Figure 3).

If the study is extended using the semi-discretization method, it can be seen that this effect is related with Hopf bifurcation and the lobe-like region is actually connected to the ordinary lobe that corresponds to point A in Figure 3. Clearly, this region is a part of an ordinary Hopf lobe and it is separated from the flip lobes. Also, ZOA is not accurate enough to capture these special areas of the linear stability charts. In fact, these interactions are more complex than the double period chatter. It changes the usual lobes introducing new shapes, reduces the location and values of the minimum stability and changes considerably the position of the most stable spindle speeds.

Traditionally, a stability diagram is formed by a number of lobes with the same value for the minimum depth of cut, but with different location of the spindle speed related to the maximum stability. Due to mode interaction, this minimum can change for lobes of different order. For instance, in the case study the minimum for the first lobe (point B) differs from the second lobe (point E) or the third one (point F).

When mode interaction happens, the chatter vibration grows with two modulated chatter frequencies close to both dominant natural frequencies ($\omega_{\mathrm{n},1}$ and $\omega_{\mathrm{n},2}$). The time domain simulations confirm this chatter frequency pattern and the stability limit (see point C in Figure 5). In other regions (point A), the harmonics are far from other modes, therefore, ZOA defines the right stability boundary.

It is important to mention that the vibration (chatter) frequencies that are calculated by multi-frequency and by semi-discretization only deal with the frequencies that appear at the stability border when the system loses its stability. Later, the vibration grows till some parts of the cutting edge lose its contact with the workpiece. Then, the dynamics changes drastically that results frequency changes and appearance of new frequencies, too (see

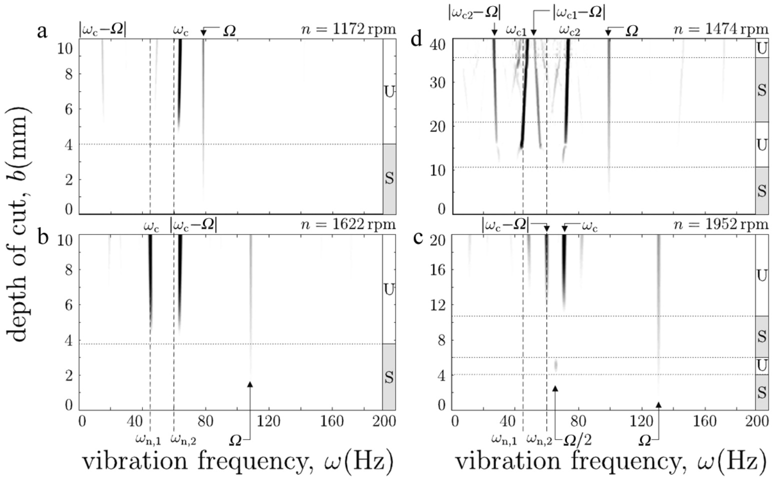


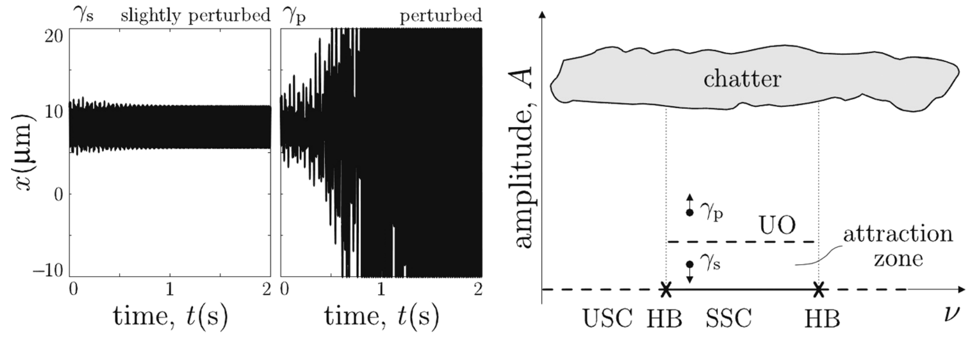
FIGURE 5 Spectra of different time-domain simulations in different spindle speeds (see points *A*, *B*, *C*, and *D* in Figure 3).

Figure 5(d)). The complexity of mode interactions increases with the number of significant modes, but the behavior is the same and the multi-frequency and semi-discretization methods are able to capture these effects.

Stable Isolated Zone

Because the semi-discretization provides information about the local behavior of the stationary cutting process, the stable and unstable areas can be identified easily. The multi-frequency solution, on the other hand, only finds the borders where the system has non-hyperbolic behavior (Guckenheimer and Holmes, 1983). Theoretically, it is impossible to point out the stable areas using MF solution only, although in machine tool vibration, one can predict that the cutting process with zero depth of cut is always stable. In this manner, the only difficulty is the identification of possible stable islands, which requires further investigations. Figure 3 shows a case, where a stable island exists, which was identified by means of semi-discretization method. Following the roots of multi-frequency in Figure 3, it is possible to realize that this stable island also appears in the results of multi-frequency solution.

The island encircled by borders is related to Hopf and flip bifurcations, which are mostly originated by the mode interaction phenomenon. Similar phenomena have been found in the literature (Munoa et al., 2009; Sellmeier and Denkena, 2011). In the case of Sellmeier and Denkena (2011), their existence has been related to the introduction of variable helix angles in the tool, whilst here, the stable island is caused by simple conventional tool and has a reachable axial depth of cut level from machining point of view.



(a) (b)

FIGURE 6 (a) Time domain simulations at point γ with different perturbation levels. (b) Bifurcation structure along ν (see Figure 3(b)). (HB: non-hyperbolic points related to Hopf bifurcation).

In fact, these isolated stables zones are theoretically the highest stable zones, and consequently the most suitable conditions to optimize production. Therefore, it is important to confirm the existence of these zones through experimental testing. However, these zones did not appear in time-domain simulations if the non-linearity related to the possibility that the tool may lose contact with the workpiece is considered when the tool starts cutting. To explain this phenomenon some control points, namely α, β, and δ (cf. Figure 3), were used checking the correctness of the linear stability and the time domain solver. However, the simulation of point γ (see Figure 6(a)) shows that the stable sense of the island does not appear in the reality.

In Figure 6 one can realize how sensitive the stable sense of the stationary solution in the isolated zone at point γ is. These solutions were simulated by using the correct, but slightly perturbed stationary cutting solution as an initial function. The results show that for small perturbation the system is attracted by the linear equilibrium, but for higher perturbation the system is pushed out and it tends to chatter. Therefore, the stable island has a little attraction zone, which means that the stable sense might vanish or be unreachable in real circumstances. This phenomenon is similar to cases addressed by Dombovari et al. (2008) or Bachrathy et al. (2011), but here the system is originally linear. Therefore, the isolated zone cannot be used in real cutting conditions to increase the material removal rate.

CONCLUSIONS

The stability of a milling process with several dominant modes has been studied using a two mode system as an example. The predictions of ZOA, multi-frequency method, semi-discretization method and time domain simulations have been compared. The precision of the ZOA is reduced when the cutting force becomes interrupted. These inaccuracies have two sources: the double period chatter and variations due to mode interaction. When the number of flutes is low, these effects appear in considerable engagements. The multi-frequency and semi-discretization methods lead to the same exact solution in all cases but the multi-frequency method has problems to determine stable isolated zones.

As it is well known, the double period chatter adds another family of lobes in the stability chart. Mathematically it is related to the flip bifurcation, and it happens when the zeroth-order term and one harmonic act on the same mode. In this case, the chatter frequency is always related to tooth passing frequency. This characteristic can help to build these lobes very fast.

Finally, the variations produced by the mode interactions have been addressed. In the frequency domain, the phenomenon is similar to the flip bifurcation, but in general it does not result in an independent family of lobes. Pure numerical methods are necessary to describe this effect; therefore, it is really difficult to develop fast prediction methods. Approximated formulas have been proposed to define the region affected by the mode interaction.

The cases considered show that the mode interaction can create isolated stable zones confirmed by different methods. However, as it was showed, this linearly stable island might not exist in real cutting processes due to the possible tiny attraction zone, concluding that any kind of perturbation can destabilize the cutting process within the theoretically stable island.

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