The effect of serration on mechanics and stability of milling cutters

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ABSTRACT

The machining behaviour of special serrated milling tools are investigated. These cutters are most commonly used for roughing operations of superalloys such as titanium and nickel based alloys which prevent high cutting speeds due to their high cutting forces and low thermal conductivity. During the experimental study, these drawbacks were avoided with the usage of aluminium alloy that allows more convenient machining circumstances and high tooth passing frequencies compared to the frequencies of the essential vibration modes. By means of a general cutting force model, simulations point out the fact that the serrated cutters require lower drive torque than their non-serrated counterparts, while our corresponding measurements validate our model. A regenerative dynamic model is constructed up directly in the modal space using the modal representation of the tool/toolholder/spindle structure and linear stability analyses are performed by the so-called semi-discretization method. The significantly larger parameter domains of stable cutting and their predicted feed dependency for these serrated mills are confirmed by chatter tests. As a result of these investigations, the practical advantages of the serrated cutters are confirmed: while they remove the same specific amount of materials using lower drive torque, their productivity can also be increased using higher stable depth of cuts compared to their non-serrated counterparts even in case of difficult-to-cut materials like titanium. The constructed mechanical model also provides an adequate tuning of the cutting parameters and the serration waves in order to optimize the process for easy-to-cut materials like aluminium.

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1. Introduction

The self-excited vibrations, chatter, are caused by the regenerative vibrations between the workpiece and tool, and they lead to poor surface finish and may even damage the machine tool and workpiece. The productivity and surface quality can be improved by identifying stable cutting conditions, designing cutters and machine tools which do not cause chatter. The pioneering stability theories of Tlusty [1] and Tobias [2] predict chatter free spindle speeds and depth of cuts by modelling the interaction between the structural vibrations and orthogonal cutting operations. The dynamics of the process was modelled by delayed differential equations (DDE) and their linear stability were solved in frequency and time domain by Tlusty and Tobias.

There has been significant advances in solving the stability of more complex cutting operations like milling in both frequency and discrete time domain. Milling dynamics are modelled by coupled, linear, or nonlinear delayed differential equations with time-periodic parameters. The stability charts of the governing DDEs can be constructed by linear stability models based either on the frequency domain (zero order solution [3], multi-frequency solution [4,5]) or on the time domain (semi-discretization [6,7], time finite element [8]). The effect of nonlinearities can be further studied analytically [9,10] or using continuation algorithms such as [11–13].

The first and most productive, chatter free stable pocket in milling with regular end mills is achieved by selecting a spindle speed that matches the dominant structural modal frequency divided by the number of teeth on the cutter. The subsequent stability pockets are located at the 2–5th integer divisions of this speed [14]. If the speed is selected at lower fractions of the dominant natural frequency, the friction between the flank of the tool and closely packed waves left on the finish surface creates process damping [15], that may also be a result of a secondary (short) regenerative effect [16]. Thermally resistant nickel and titanium alloys are however milled at the lower speeds where the productive stability pockets do not exist. It is common to use serrated end mills to disturb the regeneration of waviness at cut surfaces when roughing thermally resistant materials. The waves are ground out of phase on each flute to disturb the regeneration. The geometric modelling and an example for stability

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2. Modelling of milling with serrations

Serrated end mills have flutes with waves that are described by cubic polynomials (for 2+1/2 axis milling operations) as presented in [17]. The shape of the serration is modelled by a dimensionless function with two parameters, namely, the peak-to-peak amplitude and the wavelength. The regenerations and the missed-cuts are indicated, and the instantaneous chip thickness is derived geometrically for a general 3-axis milling case. Consequently, the feed \( f \) is always set in the \((x)\) direction as shown in Fig. 1 a.

2.1. Serration

The serration is generated by varying the local radius \( R_i \) of flute \( i \) defined by

\[
R_i(z) = R - \Delta R_i(z),
\]

where \( R \) is the radius of tool shank envelope of the cylindrical tool, and \( z \) is the axial coordinate measured from tool tip. The variation \( \Delta R_i(z) \) can be written by the dimensionless profile function \( \rho(\zeta) \)

\[
\Delta R_i(z) = a \rho \left( \frac{z}{L} \cdot \omega_i \right),
\]

where \( a \) is the peak-to-peak amplitude and the dimensionless axial coordinate \( \zeta \) is scaled with the wavelength \( L \) of the serration (Fig. 1 a,c). \( \omega_i \) indicates the phase shift of the variation on the \( i \)th flute which, in practice, a uniform division of the revolution expressed as

\[
\psi_i = \sum_{k=1}^{l-1} \phi_{p,k},
\]

where \( \phi_{p,k} \) is the pitch angle between the subsequent flutes (Fig. 1 b).

2.2. Regenerative delays between the serrated flutes

The serrated waves are ground with relative phase shift from one flute to the next, which leads to irregular regenerative phase delays during milling. Sections of the flutes do not cut when the chip load is less than the amplitude of the serrated waves on the cutting edges, which leads to doubling or tripling the effective pitch angle at the particular location. Instead of having a single phase delay as in regular end mills, i.e. one tooth passing period, the system experiences multiple time delays. The angular positions of flutes \( i \) and \( i+l \) become equal at a certain axial segment \( z \):

\[
\phi_i(z,t) = \phi_{i+l}(z,t-t_{i+l}),
\]

where the angular position of the \( i \)th flute at \( z \) has the form

\[
\phi_i(z,t) = \Omega t + \sum_{k=1}^{l-1} \phi_{p,k} - \phi_{p,l}(z). \tag{1}
\]

This angle is measured clockwise from the \((y)\) axis (Fig. 1 a,b) with three zones: the revolution part, the pitch shift part and the lag angle

\[
\phi_{p,l}(z) = \frac{z}{R} \tan \eta_i,
\]

where \( \eta \) is the helix angle of the end mill (Fig. 1 a). The time delay between the ‘actual’ \( i \)th flute and the ‘previous’ \( (i+l) \)th position

Fig. 1. (a) shows the geometry of the serrated cutter. In (b) the possible angular distances are presented between the flutes in the rotational direction (here, \( R_{i+1}(z) < R_i(z) < R_{i+1}(z) \)). (c) shows the dimensionless profile of a possible serration shape.
can be expressed as
\[ \tau_{il} = \frac{1}{N} \sum_{k=1}^{l-1} \varphi_{il} + k \bmod N. \]

The delay is the time needed to travel angle \( \varphi_{il} \) between the teeth \( i \) and \((i+l)\) at constant spindle speed \( \Omega \text{ (rad/s)} \) by the cutter (Fig. 1b). Maximum of \( N \) different delays can occur in the case of a uniform pitch serrated cutter, whereas \( N^2 - (N-1) \) amount of different constant delays are produced by serrated cutters with non-uniform pitch angles.

### 2.3. Chip thickness

The chip thickness is defined approximately as the local distance between the previously and the just cut surfaces in the direction of the local normal vector \( n_i(z) \) of the flute. The geometric chip thickness of the \( i \)th flute between the \( i \)th and the \((i+l)\)th flutes is defined as
\[ h_{gi}(z, t) = n_i(z, t), \]
which is not restricted to be positive, and does not represent the physical chip thickness. The local movement of flute \( i \) relative to flute \((i+l)\) at the same angular position is in Fig. 2a is defined by
\[ r_{il}(z, t) = r_i(z, t) - r_{i+l}(z, t - \tau_{il}), \]
where \( r_i(z, t) \) and \( r_{i+l}(z, t - \tau_{il}) \) are the vectors, in the absolute coordinate system, pointing to the tip of the edges at \( z \), while \( f_j(z, t) \) is the corresponding feed motion during \( \tau_{il} \). The regeneration between the flutes \( i \) and the \((i+l)\) is
\[ \Delta r_{il}(t) = r(t) - r(\tau_{il}). \]

Note that, the relative motion of the tool centre is given in the Cartesian coordinate system as \( r(t) = \text{col}(x(t), y(t), z(t)) \). The normal vector of the \( i \)th flute at \( z \) for uneven radii is given by
\[ n_i(z) = \begin{bmatrix} \sin \varphi_i(z) \sin \varphi_i(z) \\ \sin \varphi_i(z) \cos \varphi_i(z) \\ -\cos \varphi_i(z) \end{bmatrix}. \]

The axial immersion (lead) angle can be expressed as the spatial derivative of the radius,
\[ \cot(\kappa_i(z)) = \frac{d \varphi_i(z)}{dz}. \]

The effective geometric chip thickness can be evaluated as the minimum of the geometric chip thicknesses by taking all the past flutes into account [19].
\[ h_{gi}(z, t) = \min_{l=1}^{N} h_{gi}(z, t), \]

Since the number of the missed-cuts can change along \( z \) and may be different on each flute, the effective index and the effective delay have a simplified notation
\[ e = e_i(z, t) \quad \text{and} \quad \tau_{le} = \tau_{le,i}(z, t). \]

The physical chip thickness of the \( i \)th flute at level \( z \) can be determined as
\[ h_i(z, t) = h_{gi}(z, t) = g(z, t) h_{gi}(z, t), \]
where the switching function \( g(z, t) = g_{ei}(z, t) g_{ci}(z, t) \) is a multiplication of two different functions related to radial immersion
\[ g_{ei}(z, t) = \begin{cases} \varphi_{en} < (\varphi(z, t) - 2\pi) < \varphi_{en}, \\ 0, \quad \text{otherwise}, \end{cases} \]
and to the missed-cut effect
\[ g_{ci}(z, t) = \begin{cases} h_{gi}(z, t) > 0, \\ 0, \quad \text{otherwise}. \end{cases} \]

The entry angle \( \varphi_{en} \) and the exit angle \( \varphi_{en} \) are measured clockwise from the \( (y) \) axis (1).

### 2.4. Cutting force model

The cutting force per unit axial depth of cut (also called specific cutting force) at a particular edge location is expressed (Fig. 2b) in edge (\( \text{tra} \)) coordinate system as
\[ f_{\text{tra},i}(z, t) = -f_i h_i(z, t), \]
where \( f_i \) is the cutting force vector defined in tangential (\( t \)), radial (\( r \)) and axial (\( a \)) directions, which can be considered as a linear or nonlinear function of the chip thickness. The cutting force is assumed to have edge and cutting parts,
\[ f_i = K_e(n, z_i) + K_c(h, n, z_i, \eta) h, \]
\[ \begin{bmatrix} \mathbf{K}_e \\ \mathbf{K}_c \end{bmatrix} = \begin{bmatrix} K_{e1} & K_{e2} & K_{e3} \\ K_{c1} & K_{c2} & K_{c3} \end{bmatrix}, \]
\[ \mathbf{K}_c = \begin{bmatrix} K_{c1} & K_{c2} & K_{c3} \end{bmatrix}, \]
\[ \mathbf{K}_e = \begin{bmatrix} K_{e1} & K_{e2} & K_{e3} \end{bmatrix}, \]

Fig. 2. In (a) the possible missed-cut effect of the serrated tool is presented (the corresponding surface segments cut by different flutes mark with continuous, dashed and dots-dashed lines). (b) shows the differential forces acting on an infinitesimal \( dz \) axial segment on the \( i \)th edge. Note that \( df_{\text{tra},i} = f_{\text{tra},i} dz \).

where $K_e$ (N/m) and $K_c$ (N/m²) are the edge and cutting force coefficients, respectively [20]. The specific cutting forces are projected in feed ($x$), normal ($y$) and axial ($z$) directions as

$$ f_i(z, t) = g_i(z, t)T_i(z, t), $$

where the transformation matrix between the (tra) and (xyz) coordinate systems has the form

$$ T_i(z, t) = \begin{bmatrix} \cos \phi_i & -\sin \phi_i & \sin \alpha_i \cos \phi_i \\ \sin \phi_i & \cos \phi_i & \sin \alpha_i \sin \phi_i \\ 0 & 0 & 1 \end{bmatrix}, $$

with $\phi_i := \phi_i(z, t), \phi_c := \phi_c(z)$.

The cutting forces acting on the cutting tool is evaluated by integrating the differential forces along the flute and summing the contributions of all flutes in cut (Fig. 2 b).

$$ F(t, r_i(\theta)) = \sum_{i=1}^{N} \int_{0}^{r_i} f_i(z, t) \, dz, $$

where $r_i$ is the arc-length coordinate along the cutting edge of the corresponding flute $i$ (Fig. 2 b), and $r_i(\theta) = r_i(\theta + \theta)$ is the shift function [21] which emphasizes that force $F$ contains regeneration of the position of the tool with multiple constant delays, hence, $\theta \in [-\tau_{max}, 0]$. The force is evaluated along the axial depth of cut as

$$ F(t, r_i(\theta)) = \sum_{i=1}^{N} \int_{0}^{r_i} f_i(z, t) \, cos(\sin(z, a) / z) \, dz. $$

The integration in (6) is approximated numerically by a sum of forces at discrete points along the ($z$) axis.

3. Model validation by instantaneous force and torque measurements

The chip thickness and cutting force models are first validated experimentally in chatter free cutting tests before their incorporation to the stability model. Since the chip thickness distribution is extremely non-uniform along the cutting edges of all flutes, the classical mechanistic calibration of cutting force coefficients cannot be employed on serrated end mills. The cutting force coefficients are evaluated by orthogonal to oblique cutting transformation model described by Altintas [20]. The identified orthogonal cutting parameters of the AL7075-T6 work material contain the effects of chip size $h$ (mm), cutting speed $V_c$ (m/min) and rake angle $\alpha_r$ (deg), and given as

$$ t_s = 297.05 + 1.05z_t \text{ (MPa)}, $$

$$ \phi_r = 24.20 + 36.67h + 0.0049V_c + 0.32z_t \text{ (deg)}, $$

$$ \beta_r = 18.79 + 6.7h - 0.0076V_c + 0.2561z_t \text{ (deg)}, $$

where $\tau_s$, $\phi_r$, and $\beta_r$ are the shear stress, shear angle and friction angle, respectively. The edge-coefficient vector is given as

$$ K_e = \begin{bmatrix} 23.41 & -0.0014V_c & -0.26z_t \\ 35.16 & -0.0011V_c & -0.51z_t \\ 0 & 0 & 0 \end{bmatrix} \text{ (N/mm²)}. $$

Note that, the cutting force is a nonlinear function of the chip thickness $h$ (5).

Sample, vibration free cutting force simulation results are shown in Fig. 3 a where the resultant tangential, radial and the axial components of the cutting force are presented for serrated and regular end mills. The regular end mills produce periodic and uniform cutting forces, whereas the serrated end mills have non-uniform amplitudes at tooth passing intervals due to irregular chip load distribution among the flutes. However, the serrated end mills have a periodic pattern at spindle rotation periods as shown in Fig. 3 a (see 1–2–3–4), which is caused by the phase shifts $\psi_r$, since the cut and missed-cut ‘parts’ of the tool are different for each flute (see Fig. 3 b). Although both regular and serrated end mills remove the same amount of material per spindle revolution, the serrated end mills cut thicker chips which results in lower average cutting forces, torque and power due to size effect. The cutting force coefficients, the slopes of the cutting force curves at each chip thickness, are lower when the chip size is larger as demonstrated in Fig. 4. However, while the torque/power cost is lower for the serrated tool, the cutting edges which cut thicker chips are worn more quickly than the regular end mills.

![Image](https://via.placeholder.com/150)

**Fig. 3.** (a) presents a result of a stationary force simulation done by tool (details in Table 1) (here, $F_t$, $F_r$ and $F_a$ are the resultant tangential, radial and axial forces and $k_{s_i}=3 \text{ mm}, \, n=5000 \text{ rpm}, \, \text{half immersion down milling}$). Thin and thick lines show the forces of the serrated cutter and a conventional tool. (b) shows the chip separation process in a revolution of the serrated tool (black, light grey and dark grey mark different successive edges). The chip volumes cut by edges are depicted with greyscale corresponding to the edge-greyness (the hypothetical chip volume cut by a conventional tool is indicated by dashed frames). Note that $dV_t(z) + dV_r(z) \approx 3dV_c$. 

![Image](https://via.placeholder.com/150)

**Fig. 4.** The effect of the degressive cutting force characteristic with missed-cut effect.
Cutting tests have been conducted on a 3 axis horizontal milling machine centre using AL7075-T6 workpiece in order to test the model at a large spindle speed range and deep cuts. Titanium requires more rigid machine and the cutting speed range is limited due to tool wear constraint. However, the proposed mathematical model is valid for any metal. A short carbide tool (tool1 in Table 1) was attached to a shrink fit tool holder to increase stiffness and minimize run out. Three spatial force components \( F_x, F_y, F_z \) and the acceleration \( a_y \) perpendicular to the feed motion were collected. The acceleration signal was used to check whether the cut was stable or unstable during tests.

Three sample measurements are compared against simulated forces for regular and serrated end mills in Fig. 5a–c at different feeds per tooth. The feed rate is normalized against the peak-to-peak amplitude of the serration waves as

\[ \varepsilon = \frac{f_t}{a} = \frac{f}{Na} \]

The dimensionless feed \( \varepsilon \) indicates how the cutting force amplitudes are affected by the regular end mills as the feed increases. It can be seen that the measurements (thin lines) and the predictions (thick lines) fit well except in the \( z \) direction due to the unmodelled edges on the tool tip. The average forces \( (F_x, F_y) \) that arise at an average position angle is approximated as

\[ \overline{\sigma} = \frac{\theta_{en} + \theta_{es}}{2} \]

The corresponding average tangential force can be expressed as

\[ F_t = F_x \cos \overline{\theta} - F_y \sin \overline{\theta} \]

which is used to construct the average cutting torque as

\[ M = R F_t \]

where \( R \) is the radius of the tool envelope. In Fig. 5d, the predicted torque (thick curve) is compared with the required torque of the equivalent conventional tool (dashed line). This torque characteristics clearly imply that the serrated tool needs lower drive torque to perform the same material removal rate at low feed range than an equivalent non-serrated tool. On the other hand this difference vanishes at relatively high feed rates \( (\varepsilon \approx 1) \).

In Fig. 5d, \( \circ \)'s represent the average drive torque estimated from the force measurements described by (8)–(10).

<table>
<thead>
<tr>
<th>Table 1</th>
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<tbody>
<tr>
<td>The selected serrated tools for the force (tool1) and for the stability (tool2) measurement.</td>
</tr>
<tr>
<td>Name</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>tool1</td>
</tr>
<tr>
<td>tool2</td>
</tr>
</tbody>
</table>

Fig. 5. The first three panels show examples of the performed force measurements (the thick and thin lines are the simulated and the measured forces of the serrated cutter; the dashed lines show the simulated forces of a conventional tool without serration). One after another the presented time–force evolutions were performed with \( f_t = 0.02 \), \( 0.1 \) mm/tooth and with \( f_t = 0.25 \) mm/tooth, respectively. Panel (d) shows in percentage the predicted (continuous line) and the measured (\( \circ \)) torque characteristic of the serrated cutter (tool1). (Here, \( a_p = 3 \) mm, \( n = 5000 \) rpm, half immersion down milling.)
4. Stability of milling with serrated end mills

The process mechanics model, which is proven to be sufficiently accurate, is used in the chatter analysis of the serrated end mills. Dynamics of milling with serrated end mills is modelled by a set of delayed differential equations in the modal space. The linear stability of the process is analysed by semi-discretization (SD) method. The influence of feedrate on the stability is analysed.

4.1. Dynamics

The vibration of the tool is described by the modal coordinates \( \mathbf{q} = \text{col}(q_1, q_2, \ldots, q_n) \) and mode shape vectors \( \mathbf{p}_k \), \( k = 1, 2, \ldots, n \). The modal matrix \( \mathbf{P} = [\mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_n] \) is normalized by the modal masses \( m_k \) as

\[
\mathbf{U} = \mathbf{P} \text{diag}(\sqrt{m_k}),
\]

where the elements of the diagonal matrix are \( q_i = (m_k \mathbf{p}_k)_{1/2} \). The cutting force model is linear, one does not need to calculate the cutting force model has dependency on the chip thickness as explained in (3) and (6). The solution of the governing delayed differential equation (11) can be separated as

\[
\mathbf{q}(t) = \mathbf{q}_0(t) + \mathbf{u}(t),
\]

where the stationary solution \( \mathbf{q}_0(t) = \mathbf{q}_0(t + T) \) is a periodic function with a spindle period \( T = 2\pi/\omega \), and its perturbation \( \mathbf{u}(t) \) is considered to be small, and it is associated with the chatter. The variational system can be obtained by the linearization of the cutting force:

\[
\mathbf{F}(\mathbf{u}, \mathbf{u}(\theta)) = \sum_{j=1}^{N_c} \mathbf{H}_j(t)\mathbf{u}(t - \tau_j) - \mathbf{F}_p(t).
\]

With the substitution of (13) and (12) into (11), the variational system has the form

\[
\mathbf{u}(t) + [2\tilde{\omega}_n \mathbf{H}_j(t) + \mathbf{u}(t)] + \mathbf{H}(t)\mathbf{u}(t) = \sum_{j=1}^{N_c} \mathbf{H}_j(t)\mathbf{u}(t - \tau_j).
\]

The stability of the parametrically excited linear delayed differential equation (16) is equivalent to the stability of the periodic stationary motion \( \mathbf{q}_0(t) \).

4.2. Stability analysis with SD method

The frequency domain solutions are not suitable in this case, since the depth of cut of the serrated tool is not well defined which is used in the final solution of the constructed eigenvalue problems [3–5]. The semi-discretization technique, which considers the periodic parameters and multiple delays, is applied to determine the linear stability of the serrated cutters.

The linear variational system (16) is rewritten in first order representation, where \( \mathbf{L}(t) \) and \( \mathbf{R}(t) \) are the linear and the retarded periodic coefficient matrices, respectively:

\[
\mathbf{y}(t) = \mathbf{L}(t)\mathbf{y}(t) + \sum_{j=1}^{N_c} \mathbf{R}_j(t)\mathbf{y}(t - \tau_j), \quad \mathbf{y}(t) = \left[ \mathbf{u}(t) \right].
\]

The infinite dimensional time-periodic delayed differential equations are approximated by a series of non-homogeneous ordinary differential equations,

\[
\mathbf{y}(t) = L(t)\mathbf{y}(t) + \sum_{j=1}^{N_c} R_j(t)\mathbf{y}(t - \tau_j), \quad t \in [t_r, t_r + \Delta t],
\]

where the time-step is \( \Delta t = T/r \) with \( r \) representing the number of intervals in the period \( T \) and the time-averaged matrices are \( \mathbf{L}_n = (1/\Delta t) \int_{t_r}^{t_r + \Delta t} \mathbf{L}(t) \, dt \), \( \mathbf{R}_n = (1/\Delta t) \int_{t_r}^{t_r + \Delta t} \mathbf{R}_j(t) \, dt \). \( \sigma_n^{(p)} \) is a pth order polynomial [22] spanned by the discretized version of the state \( \mathbf{y}(t) = \mathbf{y}(t + \Delta t) \) for \( (0 \in [-\tau_{\text{max}},0]) \) of (17). The phase space is summarized in the vector

\[
\mathbf{z}_n = \text{col}(\mathbf{y}_n(0), \mathbf{y}_n(-\Delta t), \ldots, \mathbf{y}_n(-m\Delta t)),
\]

where \( \Delta t = m \Delta \theta = m \pi \Delta t/\omega \) and \( m = \text{int}(\tau_{\text{max}}/\Delta t + p/2) \). The polynomial \( \sigma_n^{(p)} \) approximates the solution \( \mathbf{y}_n(\theta) \) around \( \theta = -\Delta \theta \) (see Fig. 6). As it was shown in [22], the first order polynomial provides the most
efficient approximation:

\[ \sigma_{ij}^{(1)}(t - \tau_i) = \frac{t - \tau_i - (t - m_i) \Delta \tau}{\Delta \tau} \mathbf{y}_i(-m_i \Delta \tau) - \frac{t - \tau_i - (t + 1 - m_j) \Delta \tau}{\Delta \tau} \mathbf{y}_j(-m_j \Delta \tau), \]

where \( m_i = \text{int}(\tau_i/\Delta \tau + p/2) \), here \( p = 1 \). By means of the analytical solution of (18), a linear map can be constructed using \( \mathbf{y}(t) \) and its discrete delayed values in \( \mathbf{z} \) as an approximate initial function. The linear map provides the approximate initial function for the next time interval \( t \in [t + \Delta t, t + 2 \Delta t] \) in the form

\[ \mathbf{z}_{t+1} = \mathbf{B}_t \mathbf{z}_t, \]

where \( \mathbf{B}_t \) is the finite dimensional approximation of the infinitesimal operator generated by (17) as discussed in [23]. According to the Floquet theory [24,7] an approximate linear map \( \Phi \), which projects \( \mathbf{y}(t) \) to \( \mathbf{y}(t+\Delta t) \) after one period \( T \) of the tool rotation, can be constructed. The application of the successive mappings defined in (19) (see Fig. 6) leads to

\[ \mathbf{z}_{t+T} = \Phi \mathbf{z}_t, \]

where \( \Phi \) is the transition matrix and its (complex) eigenvalues are the characteristic multipliers \( \mu_i \). The variational system (16), which is needed to rearrange the different parts of the resultant force (6) according to the delays \( \tau_{ij} \) occurring in the dynamic model of the serrated tools, is (11) orbitally asymptotically stable if all the characteristic multipliers are in modulus less than one. If there exists a characteristic multiplier outside the unit circle of the complex plane, the system is unstable, and chatter occurs.

During the calculation of the characteristic multipliers, only the linear map \( \Phi \) is determined numerically and the actual vibration is not traced. Consequently the original form (3) of the actual chip thickness is not suitable to determine the possible missed-cuts due to the serrations. Instead of the instantaneous geometrical chip thickness \( h_{\text{st},i}(z,t) \), one can use its static part only, which has the form (see (2))

\[ h_{\text{st},i}(z,t) = (R_i(z) - R_{i-1}(z)) \sin k_i(z) + f_i(z) \sin k_i(z) \sin \varphi_i(z,t), \]

and the effective index \( c \) can be determined from

\[ h_{\text{st},i}(z,t; c_{eff}) = \min_{\Delta \tau} h_{\text{st},i}(z,t), \]

which is needed to rearrange the different parts of the resultant force (6) according to the delays \( \tau_{ij} \) occurring in the dynamic model of the serrated tools. The missed-cut part of the switching function \( g_{\text{st},i} \) in (4) has to be modified similarly to (20).

4.3. Effect of feedrate on stability

A set of simulated stability charts is given for serrated and regular end mills in Fig. 7 at different feeds. The tool has \( N = 3 \) serrated helical flutes, see tool2 in Table 1 for the geometric parameters. The dynamic parameters of the tool/toolholder/spindle structure are experimentally identified and listed in Table 2. The stability charts of the three-fluted serrated tools

Table 2 The measured modal parameters (natural frequency \( f_{\text{Nat}} \), damping factor \( \zeta_i \) and modal stiffness \( k_i \) of the corresponding \( k \)-th mode \( k = 1, 2, \ldots, 5 \)) and the modal matrix.

<table>
<thead>
<tr>
<th>( k )</th>
<th>( f_{\text{Nat}} ) (Hz)</th>
<th>( \zeta_i ) (%)</th>
<th>( k_i ) (N/\mu m)</th>
<th>Modal matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>667.87</td>
<td>3.88</td>
<td>18.244</td>
<td>( P' = \begin{bmatrix} 1 &amp; 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 1 &amp; 1 &amp; 1 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \end{bmatrix} )</td>
</tr>
<tr>
<td>2</td>
<td>995.21</td>
<td>3.44</td>
<td>12.834</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>639.95</td>
<td>6.43</td>
<td>11.715</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>922.78</td>
<td>2.40</td>
<td>19.612</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1110.6</td>
<td>6.00</td>
<td>18.509</td>
<td></td>
</tr>
</tbody>
</table>
(thick lines) are compared to its equivalent three and one fluted non-serrated counterparts (thin and dashed lines, respectively). It can be observed in Fig. 7 that the serrated cutter behaves like an equivalent one fluted non-serrated tool at low feed rates (the thick and dashed lines almost cover each other). However, when the feed rate is increased, the stability limits of the serrated three-fluted tool gradually approaches the stability limits of the equivalent non-serrated three-fluted tool (Fig. 7 b–f). Since the higher the feed rate is, the more portions of the edges participate in cutting, and the missed-cuts stop occurring, the tool starts behaving like its non-serrated end mill (Fig. 8).

The operation can be characterized with an equivalent non-integer flute number:

\[ N = N - 1. \] (21)

The missed-cuts are averaged along the edges for a time period \( T \),

\[ \bar{\tau} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{m_i} \int_{\mu_i}^{\mu_{i+1}} e_i(z,t) \, d\mu, \]

where

\[ m_i = \mu_i(\{z,t\} \in [0,a] \times [0,T]| g(z,t) > 0) \]

is a measure defined above a theoretical surface spanned by the axial direction of the tool and the time period where the edges are in cut.

Globally, the minimum of the stability limit is shifted downwards, and the system approaches to the stability of the

![Fig. 8. The measurement set-up with accelerometers and with the minimum quantity lubrication (MQL) system.](image)

![Fig. 9. The first three panels present the result of a chatter test performed by serrated tool 2 using different feeds (○ and • mark stable stationary cutting and cutting subjected to chatter vibration). Panels (d)–(f) show a measurement sample signed with an arrow in the stability charts. The measured acceleration signal in (e) direction is presented in panel (d); its integrated FFT is in panel (e) and the surface left by the cutter is presented in panel (f). Besides the feed dependency of the stability the typical surface pattern left by an unstable tool can be recognized at the last part of (f). The measurements were performed under half immersion down milling.](image)
conventional three-fluted cutter with an equivalent number of flutes given in (21) [20].

4.4. Chatter experiments

The proposed stability model is experimentally verified in milling AL7075-T6 workpiece material at a wide range of cutting conditions using uncoated tool2 with three flutes (Table 1). The tool was placed in a shrink fit holder with a large overhang (Lc=105 mm). The measured frequency response functions (FRFs) were subjected to a rational fraction polynomial fitting method, the identified modal parameters are given in Table 2.

The milling tests have been conducted on a horizontal milling machine with minimum quantity lubrication (MQL) to avoid build up edge. The feed dependency of the stability is investigated by conducting cutting tests at three different feeds (f1=0.1 mm/tooth, f2=0.2 mm/tooth, f3=0.3 mm/tooth). The travel distance in each feed zone corresponds to about 500 spindle rotations, thus the zone lengths are different as can be distinguished in the photo shot in Fig. 9f. The depth of cut a0 was increased step-by-step starting below the predicted stability limits (Fig. 7) until the chatter limit was reached during the experiments. Sound pressure, and the accelerations of the spindle were measured in the feed and the normal directions.

Chatter was identified by the recognition of increasing characteristic peaks (see Fig. 9d) in the spectrum of the acceleration and sound signals, and by the chatter marks left on the cut surface of the workpiece (see Fig. 9f).

Chatter frequencies are presented in Hz at the different cutting speeds in Fig. 9a-c, which are close to the measured modes given in Table 2 except the case of the right-most experiment marked with PD. This type of stability loss is related to period doubling (PD, also called flip) in milling [5,25–27]. However, the prediction of this period doubling requires more sophisticated mechanical modelling where the switching between constant delays depends on the vibration itself (compare the chip thickness representations in (3) and (20)). This may lead to the use of state-dependent delay models similar to the ones in [12,28]. The predicted and measured stability limits are in sufficient agreement as demonstrated in Fig. 9.

5. Conclusion

The mechanics, dynamics and stability models of milling with serrated cutters are investigated in this paper. The serrated cutting edges are ground with phase shifts on each flute, which creates non-uniform chip generation geometry along each flute as well as among the flutes. The process is periodic at spindle rotation intervals, but the time delay at any point along the cutting edge may differ due to serration geometry, amplitude of the serration wave and feed-rate. The dynamics of the process are therefore modelled by a set of delayed differential equations with multiple delays and time-periodic parameters. The stability of the system is solved using semi-discretization method.

It is shown that when the chip thickness is smaller than the amplitude of the serration waves, part of the flutes do not have a contact with the material. As a result, the number of delays increases and the serrations attenuate the regeneration and the stability limits increase. If the feedrate increases, the material contact along the serrated flutes increases, the stability limits are reduced and the machining process approaches to the performance of regular, smooth end mills. As a rule of thumb, the serrated cutters produce approximately the number of flutes (Nv) times higher axial depth of cuts than the regular end mills provided that the feed rate is less than the peak-to-peak amplitude of serration waves ground on the cutter. This effect is opposite to the behaviour caused by the nonlinear cutting force characteristics in conventional milling, which tends to destabilize the cutting process at low feed ranges. In the meantime, serrated cutters behave as one fluted regular end mills at low feed ranges, that means, the possible non-uniform pitch angles have no effect on the stability. Additional constraints, such as serrated edge failure and flank wear caused by large chip loads need to be considered in selecting the productive but efficient feeds when serrated end mills are used in roughing difficult to machine alloys.

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References


