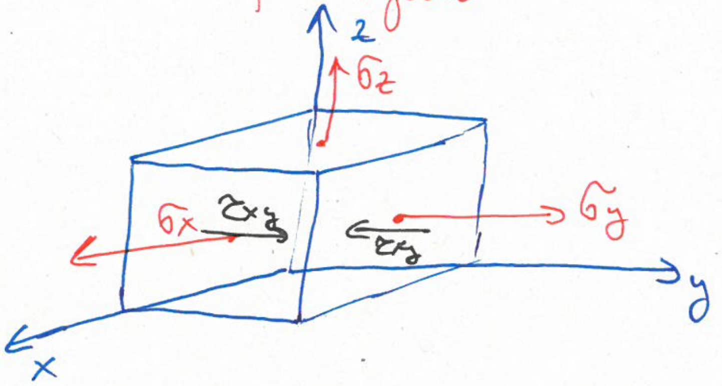
1. Feladat

Egy nyugalmas test valamely pontjában az alábbi feszültségek ébrednek

$$\underset{(x,y,z)}{\underline{\underline{\sigma}}} = \begin{bmatrix} 80 & 60 & 0 \\ 60 & 30 & 0 \\ 0 & 0 & 20 \end{bmatrix} \text{MPa}$$

Határozzuk meg Mohr körök segítségével a főfeszültségeket és a főirányokat!



__Biztos:__

$\sigma_z = 20$ MPa főfeszültség (sajátérték)

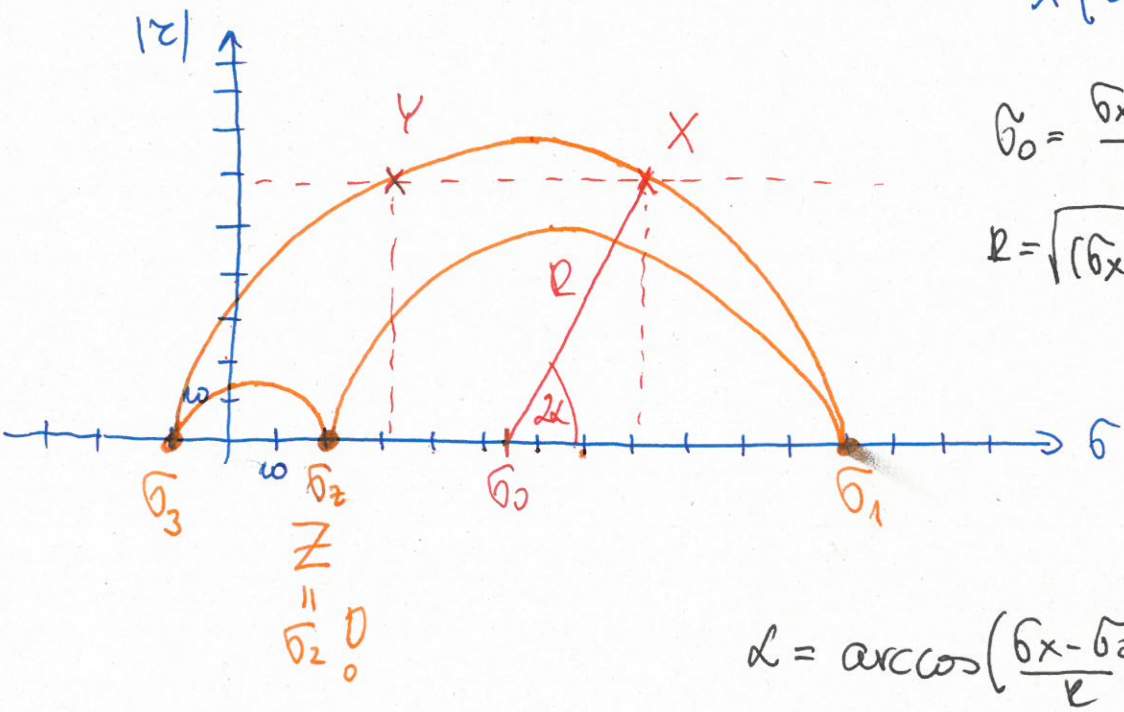
↳ a hozzá tartozó irány (főirány)

$$e_z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Mohr körök

① Felveszem az ismert pontokat

$X(80;60)$; $Y(30,60)$



$$\sigma_0 = \frac{\sigma_x + \sigma_y}{2} = 55 \text{ MPa}$$

$$R = \sqrt{(\sigma_x - \sigma_0)^2 + \tau_{xy}^2} = \sqrt{25^2 + 60^2}$$

$$R = 65 \text{ MPa}$$

$$\sigma_1 = \sigma_0 + R = 120 \text{ MPa}$$

$$\sigma_2 = \sigma_z = 20 \text{ MPa}$$

$$\sigma_3 = \sigma_0 - R = -10 \text{ MPa}$$

$$\alpha = \frac{\arccos\left(\frac{\sigma_x - \sigma_0}{R}\right)}{2} = 33{,}69^\circ$$

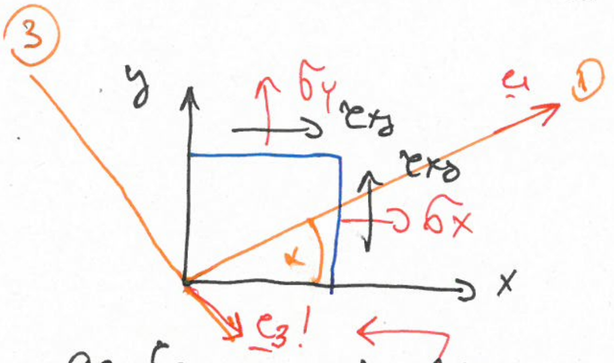
Tehát a főfeszültségek

$\sigma_1 = 120\ \text{MPa} \rightarrow \underline{e}_1 = (\cos\alpha, \sin\alpha, 0)$

$\sigma_2 = 20\ \text{MPa} \rightarrow \underline{e}_2 = (0, 0, 1)$

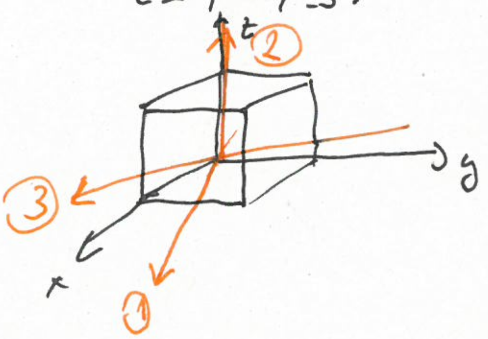
$\sigma_3 = -10\ \text{MPa} \rightarrow \underline{e}_3 = (\sin\alpha, -\cos\alpha\ \ 0)$

előtte!



Az ismert főirányból tekintek rá!

az $\{\underline{e}_1, \underline{e}_2, \underline{e}_3\}$ bázis rendszernek jobbsodrásúnak kell lennie!

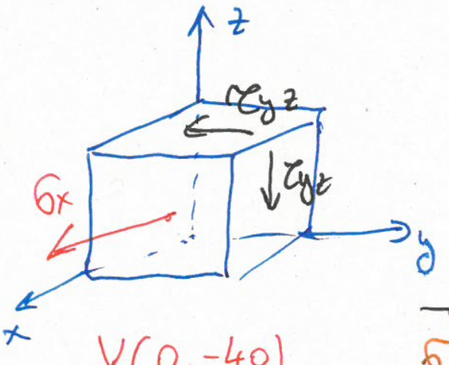


$$\underline{e}_1 = \begin{bmatrix} 0,83205 \\ 0,5547 \\ 0 \end{bmatrix};\quad \underline{e}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix};\quad \underline{e}_3 = \begin{bmatrix} 0,5547 \\ -0,83205 \\ 0 \end{bmatrix}$$

2. Feladat

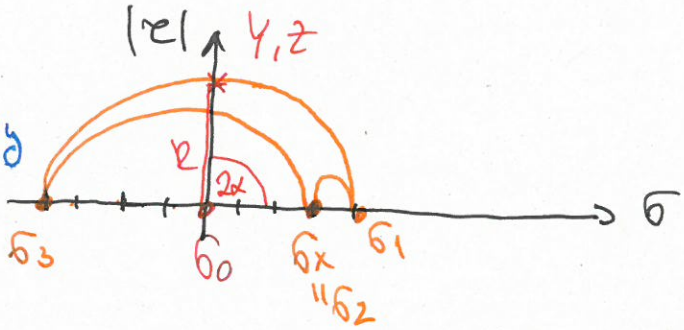
$$\underline{\underline{\sigma}} = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 0 & -40 \\ 0 & -40 & 0 \end{bmatrix} \text{MPa} \Longrightarrow \sigma_x = 30\ \text{MPa főfesz.}$$

$\underline{e}_x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ főirány



$Y(0,-40)$

$Z(0,-40)$



$$\sigma_0 = \frac{\sigma_y + \sigma_z}{2} = 0\ \text{MPa}$$

$$R = \sqrt{(\sigma_y - \sigma_0)^2 + \tau_{yz}^2} = \sqrt{0^2 + 40^2} = 40\ \text{MPa}$$

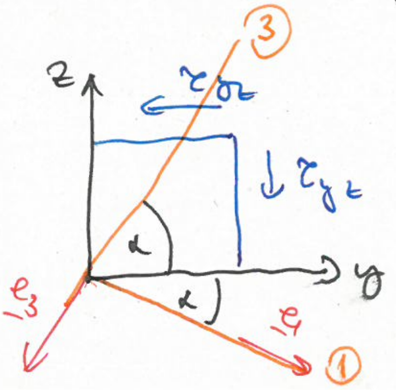
$2\alpha = 90°$

$\underline{\underline{\alpha = 45°}}$

$\sigma_1 = \sigma_0 + R = 40\ \text{MPa}$

$\sigma_2 = 30\ \text{MPa}$

$\sigma_3 = \sigma_0 - R = -40\ \text{MPa}$

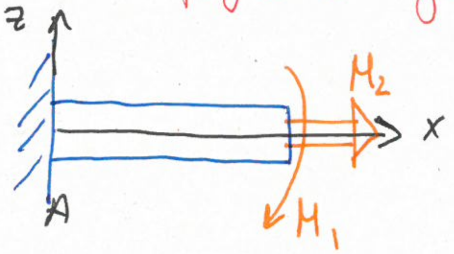


$$\underline{e}_1 = \begin{bmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix};\quad \underline{e}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix};\quad \underline{e}_3 = \begin{bmatrix} 0 \\ -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix}$$

Jobbkézszabály teljesüljön!

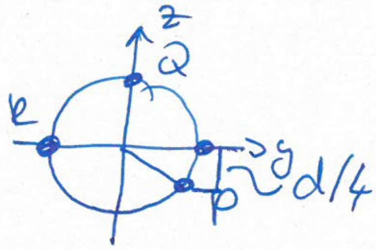
3. Feladat

Ábrázoljuk a P, Q, R pontokban a feszültségeket kiskockán, írjuk fel a feszültségi tenzor mátrixának elemeit és Mohr körök segítségével határozzuk meg a főirányokat és a főfeszültségeket!



$M_1 = M_2 = 530\ \mathrm{Nm}$

$d = 30\ \mathrm{mm}$

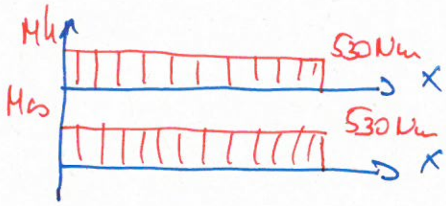


Reakciók

$M_{tA} = 530\ \mathrm{Nm}$

$M_{hA} = 530\ \mathrm{Nm}$

Igénybevételek

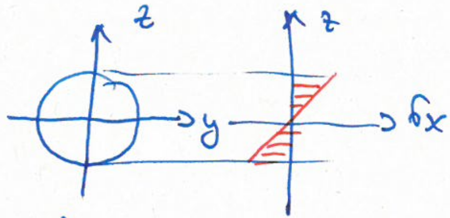


Geometria $I_y = \dfrac{d^4\pi}{64} = 39760{,}8\ \mathrm{mm}^4$

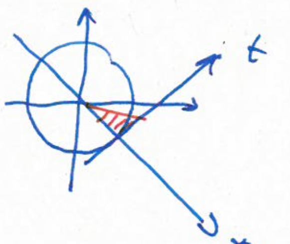
$I_p = \dfrac{d^4\pi}{32} = 79521{,}6\ \mathrm{mm}^4$

P- pont

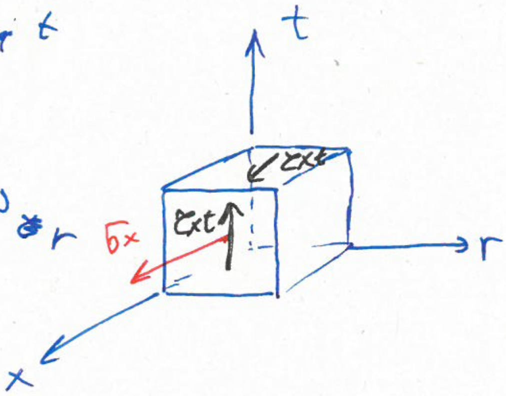
$$\sigma_x = \frac{M_1}{I_y}\cdot\left(-\frac{d}{4}\right) = -99{,}97\ \mathrm{MPa}$$



$$\tau_{xt} = \frac{M_t}{I_p}\cdot\frac{d}{2} = 99{,}97\ \mathrm{MPa}$$

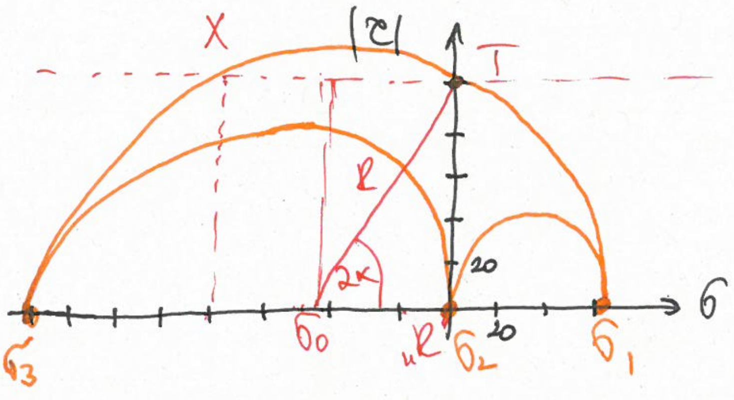


$$\underline{\underline{\sigma}}_{(x,r,t)} = \begin{bmatrix} -99{,}97 & 0 & 99{,}97 \\ 0 & & \\ 99{,}97 & 0 & 0 \end{bmatrix} \mathrm{MPa}$$



↳ $\sigma_r = 0\ \mathrm{MPa}$ főfesz! $\underline{e_r} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ főirány

$T(0,\ 99{,}97)$



$$\sigma_0 = \frac{\sigma_x + \sigma_T}{2} = -50\ \mathrm{MPa}$$

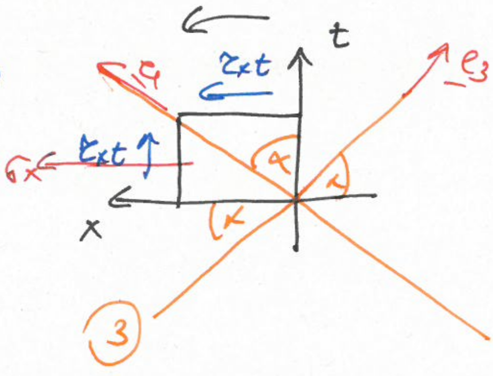
$$R = \sqrt{50^2 + 100^2} = 111{,}803\ \mathrm{MPa}$$

$\sigma_1 = \sigma_0 + R = 61{,}803\ \mathrm{MPa}$

$\sigma_2 = 0\ \mathrm{MPa}$

$\sigma_3 = \sigma_0 - R = -161{,}803\ \mathrm{MPa}$

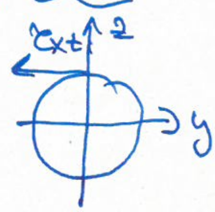
$$\alpha = \arccos\frac{\left(\frac{\sigma_t - \sigma_0}{R}\right)}{2} = 31{,}72^\circ$$



$$\underline{e}_1 = \begin{bmatrix} -\sin\alpha \\ 0 \\ \cos\alpha \end{bmatrix} = \begin{bmatrix} -0{,}5257 \\ 0 \\ 0{,}8506 \end{bmatrix}$$

$$\underline{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; \quad \underline{e}_3 = \begin{bmatrix} \cos\alpha \\ 0 \\ \sin\alpha \end{bmatrix} = \begin{bmatrix} 0{,}8506 \\ 0 \\ 0{,}5257 \end{bmatrix}$$

Q pont

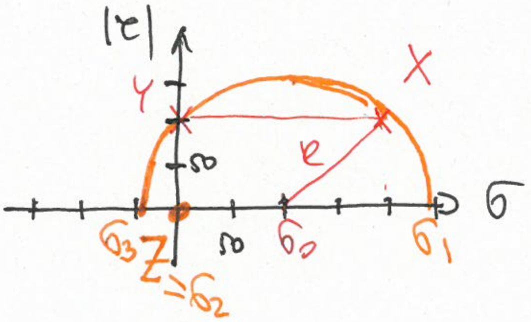


$$\sigma_x = \frac{M_h}{I_y}\cdot\frac{d}{2} = 199{,}95 \approx 200\ \text{MPa}$$

$$\tau_{xy} = \frac{M_t}{I_p}\cdot\frac{d}{2} = -100\ \text{MPa}\ !$$

$$\underset{(x,y,z)}{\underline{\underline{\sigma}}} = \underset{(x,r,t)}{\underline{\underline{\sigma}}} = \begin{bmatrix} 200 & -100 & 0 \\ -100 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \sigma_z = 0\ \text{MPa főfesz.}$$

$$\underline{e}_y = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ főir.}$$



Y(0,-100)
X(200,100)

$$\sigma_0 = \frac{\sigma_x + \sigma_y}{2} = 100\ \text{MPa}$$

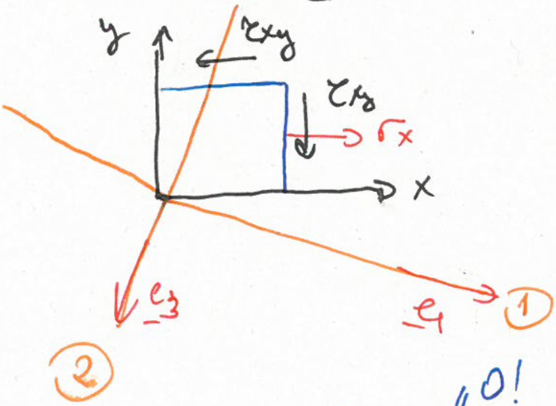
$$R = \sqrt{(\sigma_x - \sigma_0)^2 + \tau_{xy}^2} = 141{,}4\ \text{MPa}$$

$$\sigma_1 = \sigma_0 + R = 241{,}421\ \text{MPa}$$

$$\sigma_2 = 0\ \text{MPa}$$

$$\sigma_3 = \sigma_0 - R = -41{,}421\ \text{MPa}$$

$$\alpha = \arccos\frac{\left(\frac{\sigma_x - \sigma_0}{R}\right)}{2} = 22{,}5^\circ$$

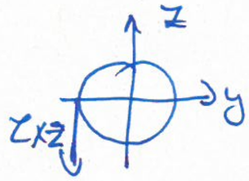


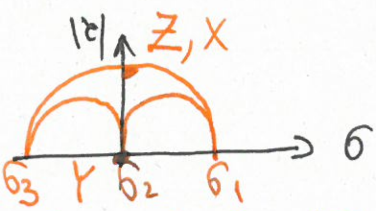
$$\underline{e}_1 = \begin{bmatrix} \cos\alpha \\ -\sin\alpha \\ 0 \end{bmatrix} = \begin{bmatrix} 0{,}9239 \\ -0{,}3827 \\ 0 \end{bmatrix}$$

$$\underline{e}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad \underline{e}_3 = \begin{bmatrix} -\sin\alpha \\ -\cos\alpha \\ 0 \end{bmatrix} = \begin{bmatrix} -0{,}3827 \\ -0{,}9239 \\ 0 \end{bmatrix}$$

R pont

$$\sigma_x = \frac{M_h}{I_y}\cdot(\overset{=0!}{z}) = \underline{0\ \text{MPa}}$$





$$\underset{(x,y,z)}{\underline{\underline{\sigma}}} = \begin{bmatrix} 0 & 0 & -100 \\ 0 & 0 & 0 \\ -100 & 0 & 0 \end{bmatrix} \rightarrow \sigma_y = 0\ \text{MPa}, \quad \underline{e}_y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\sigma_0 = 0\ \text{MPa}$$

$$\sigma_1 = 100\ \text{MPa}; \quad \sigma_2 = 0\ \text{MPa}, \quad \sigma_3 = -100\ \text{MPa}$$

$$\alpha = 45^\circ$$