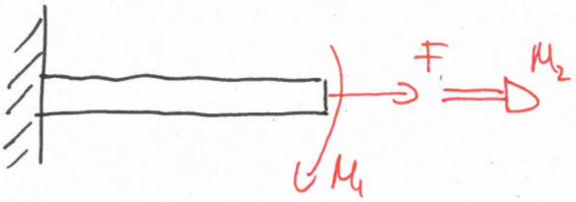
$\boxed{\text{4. feladat}}$ Egy $l = 1\,m$ hosszú, $d = 10\,mm$ átmérőjű rúd végén $F = 1\,kN$ húzóerő, M_1 hajlító nyomaték és M_2 csavarónyomaték működik. Mekkora legyen M_1 és M_2, ha azt akarjuk, hogy az egyes terhelések esetében ugyanakkora legyen az alakvált E a rúdban? Mekkora a teljes alakvált $E = ?$



$E = 200\,GPa$

$d = 10\,mm$

$\nu = 0{,}3$

Húzóerőből:

$$U^N = \int_l \frac{N^2(x)}{2AE}\,dx = \int_0^l \frac{F^2}{2AE}\,dx = \frac{F^2 l}{2\cdot\frac{d^2\pi}{4}E} = \underline{\underline{\frac{2F^2 l}{d^2\pi E}}}$$

Hajlításból

$$U^{Mh} = \int_l \frac{M_h^2}{2I_yE}\,dx = \int_0^l \frac{M_1^2}{2I_yE}\,dx = \frac{M_1^2 l}{2I_yE} = \frac{M_1^2 l}{2\cdot\frac{d^4\pi}{64}E} = \frac{32M_1^2 l}{d^4\pi E}$$

$$\text{ha } \frac{U^N}{U^{Mh}} = 1 \rightarrow \frac{\cancel{2}F^2\cancel{l}}{d^2\cancel{\pi}\cancel{E}} = \frac{\overset{16}{\cancel{32}}M_1^2\cancel{l}}{d^4\cancel{\pi}\cancel{E}} \rightarrow F^2 = \frac{16M_1^2}{d^2}$$

$$M_1^2 = \frac{d^2F^2}{16}$$

$$\boxed{M_1 = \frac{F\cdot d}{4}}$$

Csavarásból.

$$U^{Mcs} = \int_l \frac{M_{cs}^2}{2I_pG}\,dx = \int_0^l \frac{M_{cs}^2}{2I_pG}\,dx = \frac{M_{cs}^2 l}{2I_pG} = \frac{M_{cs}^2 l}{2\frac{d^4\pi}{32}\cdot\frac{E}{2(1+\nu)}} = \frac{32M_{cs}^2 l(1+\nu)}{d^4\pi E}$$

$U^N = U^{M_{cs}}$

$$\frac{2F^2\ell}{\cancel{d^2\pi E}} = \frac{\overset{16}{\cancel{32}}\, M_{cs}^2\, \ell(1+\nu)}{\cancel{d^4\pi E}} \longrightarrow M_{cs}^2 = \frac{F^2 \cdot d^2}{16(1+\nu)}$$

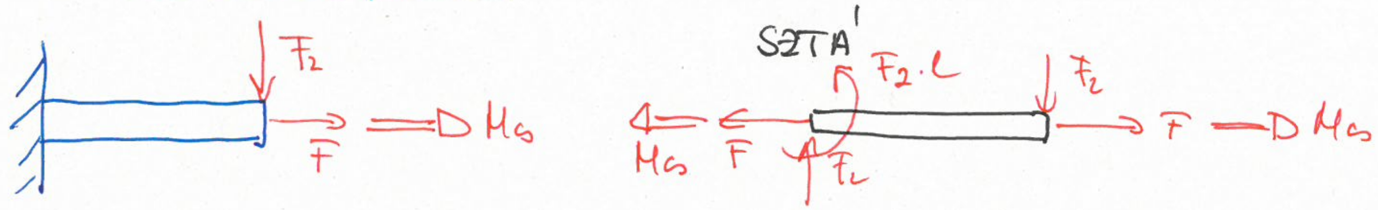
$$M_{cs} = \frac{F\cdot d}{4}\cdot\sqrt{\frac{1}{1+\nu}}$$

Numerikusan:

$$U^N = U^{M_h} = U^{M_{cs}} = 3,183\ \text{Nmm} = 3,183\cdot 10^{-3}\ \text{J}$$

$$U = 3U^N = 9,5\cdot 10^{-3}\ \text{J}$$

Módosítsuk a példát.



$$M_h(x) = F_2\cdot\ell - F_2 x$$

$$U^{M_{h_2}} = \frac{1}{2I_yE}\int_0^{\ell}(F_2\ell - F_2x)^2dx = \frac{1}{2I_yE}\left[\frac{(F_2\ell - F_2x)^3}{3(-F_2)}\right]_0^{\ell} =$$

$$= \frac{1}{2I_yE}\left(-\frac{F_2^3\ell^3}{3(-F_2)}\right) = \frac{1}{2I_yE}\,\frac{F_2^2\ell^3}{3} = \frac{32F_2^2\ell^3}{3d^4\pi E}$$

$U^{M_{h_2}} = U^N$

$$\frac{\overset{16}{\cancel{32}}\,F_2^2\,\ell^{\cancel{3}\,2}}{3\cancel{d^4}^{\,2}\cancel{\pi E}} = \frac{\cancel{2}F^2\cancel{\ell}}{\cancel{d^2\pi E}} \qquad F_2^2 = \frac{F^2\cdot 3d^2}{16\ell^2}$$

$$\Downarrow$$

$$F_2 = \frac{F\cdot d}{4\ell}\sqrt{3}$$

Elméleti összefoglaló

Az $\underline{\underline{A}}$ mátrix felbontható két mátrix összegére:

$$\underline{\underline{A}} = \underline{\underline{A}}_d + \underline{\underline{A}}_g$$

$\underline{\underline{A}}_d$ – deviátoros

$\underline{\underline{A}}_g$ – gömbi / hidrosztatikus

$$\underline{\underline{A}}_g = \frac{1}{3} \mathrm{Tr}(\underline{\underline{A}}) = \frac{1}{3} A_I \underline{\underline{E}} = \frac{1}{3} (A_x + A_y + A_z) \underline{\underline{E}}$$

↳ diagonális!

$$\underline{\underline{A}}_d = \underline{\underline{A}} - \underline{\underline{A}}_g = \underline{\underline{A}} - \frac{1}{3} A_I \underline{\underline{E}}$$

↳ főátlóbeli elemek összege zérus!

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}_{g/H} + \underline{\underline{\sigma}}_d$$

$$\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}_{g/H} + \underline{\underline{\varepsilon}}_d$$

hasonlóan

↓

$$U = U_d + U_g$$

$$u_d = \frac{1}{2} \underline{\underline{\sigma}}_d : \underline{\underline{\varepsilon}}_d$$

$$u_g = \frac{1}{2} \underline{\underline{\sigma}}_g : \underline{\underline{\varepsilon}}_g = \frac{1}{2} \underline{\underline{\sigma}}_H : \underline{\underline{\varepsilon}}_H$$

5. feladat

Egy lágyacélból készített test valamely pontjában egytengelyű a fesz. állapot. A vonzásos főfeszültség σ. Anyagjellemzők E és ν.

Mekkora az alakváltozási E-sűrűség.

Mekkora a térfogatváltozásra / torzulásra jutó rész?

$\sigma = 100$ MPa

$E = 200$ GPa

$\nu = 0{,}3$

$$\underset{(x,y,z)}{\underline{\underline{\sigma}}} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \sigma_I = \sigma = \underline{\underline{100 \text{ MPa}}}$$

$$\underset{(x,y,z)}{\underline{\underline{\varepsilon}}} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & -\nu\varepsilon & 0 \\ 0 & 0 & -\nu\varepsilon \end{bmatrix}$$

$$\underline{\underline{\sigma}}_H = \frac{1}{3}\sigma_I \underline{\underline{E}} = \begin{bmatrix} 1/3\,\sigma & 0 & 0 \\ 0 & 1/3\,\sigma & 0 \\ 0 & 0 & 1/3\,\sigma \end{bmatrix}$$

$$\underline{\underline{\sigma}}_d = \underline{\underline{\sigma}} - \underline{\underline{\sigma}}_H = \begin{bmatrix} 2/3\,\sigma & 0 & 0 \\ 0 & -1/3\,\sigma & 0 \\ 0 & 0 & -1/3\,\sigma \end{bmatrix}$$

$$u = \frac{1}{2}\underline{\underline{\sigma}} : \underline{\underline{\varepsilon}} = \frac{1}{2}\underline{\underline{\sigma}} : \left[\frac{1+\nu}{E}\left(\underline{\underline{\sigma}} - \frac{\nu}{1+\nu}\sigma_I \underline{\underline{E}}\right)\right]$$

$$= \frac{1}{2}\frac{1+\nu}{E}\underline{\underline{\sigma}} : \underline{\underline{\sigma}} + \frac{1}{2}\frac{1+\nu}{E}\frac{(-\nu)}{1+\nu}\sigma_I \underbrace{\underline{\underline{\sigma}} : \underline{\underline{E}}}_{\sigma_I} = \underline{\underline{\frac{1}{4G}\left(\underline{\underline{\sigma}} : \underline{\underline{\sigma}} - \frac{\nu}{1+\nu}\sigma_I^2\right)}}$$

$$\frac{2(1+\nu)}{4E} = \frac{1}{4G}$$

Nálunk:

$$u = \frac{2(1+\nu)}{4E}\left(\sigma^2 - \frac{\nu}{1+\nu}\sigma^2\right) = \frac{\cancel{2(1+\nu)}}{\cancel{2}4E}\frac{1}{\cancel{(1+\nu)}}\sigma^2 = \underline{\underline{\frac{1}{2}\frac{\sigma^2}{E}}}$$

$$\underline{\underline{u = 2{,}5 \cdot 10^{-4}\ \mathrm{J/cm^3}}}$$

$$\underline{\underline{u = u_g + u_d}}$$

Gömbi | Hidrosztatikus

$$u_g = \frac{1}{2}\,\underline{\underline{\sigma}}_g : \underline{\underline{\varepsilon}}_g = \frac{1}{2}\,\underline{\underline{\sigma}}_g : \frac{1+\nu}{E}\left(\underline{\underline{\sigma}}_g - \frac{\nu}{1+\nu}\,\sigma_I\,\underline{\underline{E}}_g\right)$$

$$u_g = \frac{1}{2}\,\frac{1+\nu}{E}\left(\underbrace{\underline{\underline{\sigma}}_g : \underline{\underline{\sigma}}_g}_{} - \frac{\nu}{1+\nu}\,\sigma_I\,\underbrace{\underline{\underline{\sigma}}_g : \underline{\underline{E}}_g}_{\frac{1}{3}\sigma_I\,\underbrace{\underline{\underline{E}} : \underline{\underline{E}}_g}_{3}}\right)$$

$$\frac{1}{3}\sigma_I\,\underline{\underline{E}} : \frac{1}{3}\sigma_I\,\underline{\underline{E}}$$

$$\left(\frac{1}{3}\right)^2\sigma_I^2\,\underbrace{\underline{\underline{E}} : \underline{\underline{E}}}_{3} = \frac{1}{3}\sigma_I^2$$

$$u_g = \frac{1}{2}\,\frac{1+\nu}{E}\,\frac{\sigma_I^2}{3}\left(1 - 3\,\frac{\nu}{1+\nu}\right) = \frac{1}{2}\,\frac{\cancel{1+\nu}}{E}\,\frac{\sigma_I^2}{3}\,\frac{1-2\nu}{\cancel{1+\nu}}$$

$$\boxed{u_g = \frac{1-2\nu}{6E}\,\sigma_I^2}$$

Nálunk: $\underline{\underline{u_g = 3{,}3\cdot 10^{-5}\ J/cm^3}}$

Deviátoros

$$u_d = \frac{1}{2}\,\underline{\underline{\sigma}}_d : \underline{\underline{\varepsilon}}_d = \frac{1}{2}\,\underline{\underline{\sigma}}_d : \left(\frac{1+\nu}{E}\left(\underline{\underline{\sigma}}_d - \frac{\nu}{1+\nu}\,\sigma_I\,\underbrace{\underline{\underline{E}}_d}_{=0!}\right)\right) =$$

$$= \frac{1}{2}\,\frac{1+\nu}{E}\,\underline{\underline{\sigma}}_d : \underline{\underline{\sigma}}_d = \underline{\underline{\frac{1}{4G}\,\underline{\underline{\sigma}}_d : \underline{\underline{\sigma}}_d}}$$

Nálunk:

$$u_d = \frac{1+\nu}{2E}\left(\left(\frac{2}{3}\sigma\right)^2 + \left(-\frac{1}{3}\sigma\right)^2 + \left(-\frac{1}{3}\sigma\right)^2\right) = \frac{1+\nu}{2E}\,\sigma^2\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) =$$

$$= \frac{1+\nu}{\cancel{2}E}\,\sigma^2\,\frac{\cancel{6}^{\,3}}{\cancel{9}_{\,3}} = \underline{\underline{\frac{1+\nu}{3E}}}\,\sigma^2 = 2{,}2\cdot 10^{-4}\ \underline{\underline{J/cm^3}}$$