Computation of the Lower Envelope of Stability Lobes

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Abstract

Computation of the stability limits of cutting operations is essential in order to optimize the machining process to increase productivity. Most of the computational methods consider the exact model of the system and do not take into account any uncertainties. However, the stability charts are highly sensitive to the change of the natural frequency and the spindle speed. For machining operations at low spindle speeds, the computation of the stability boundaries requires very high computational effort and unnecessarily high resolution. In these cases, the accurate boundaries are of no interest from practical point of view, thus the computation of the lower envelope of the lobe curves might be sufficient. An algorithm has been developed to determine the robust stability limits of delayed dynamical systems, which define the lower envelope curve of the stability lobes. This robust stability limit is not sensitive to fluctuations of selected parameters of the mechanical system. The algorithm is combined with the efficient Multi-Dimensional Bisection Method. A one-degree-of-freedom orthogonal turning process with process damping is investigated first, then a multi-degree-of-freedom system, the two-cutter turning model with process damping is analysed.

Keywords:

robust stability, chatter, time delay

1 INTRODUCTION

In order to reach high process efficiency the material removal rate (MRR) of cutting has to be maximized. The major limitations for increasing the MRR are the so-called chatter vibrations.

The origin of these self-excited vibrations is the surface regeneration effect, which can be described by linear delayed differential equations (DDEs) [1,2].

The stability properties are usually presented in the form of the so-called stability chart that identifies those ranges of parameters where the linear system is stable.

Many well-known computational techniques are available to determine the stability chart of DDEs [3,4,5,6,7,8].

In many cases, especially for machining operations at low spindle speed, the computation of the stability boundary requires very high computational effort and unnecessarily high resolution, due to the dense and sharp line segments of the stability boundary. In these cases, the computation of the lower envelope of the dense stability lobe structure would be adequate.

In case of time-domain computations [4,7,9,10], a really high degree of discretization is required to obtain appropriate results. In frequency domain computations [8,11,12,13], the determination of the very dense lobe structure in the unstable domain is unnecessary and also high resolution is required to determine the chatter frequency parameter, which is also an essential computational step.

Furthermore, these computational methods consider exact models of the system, and do not take into account the uncertainty of the input parameters; nevertheless, the results of these computations are highly sensitive to the change of input parameters such as eigenfrequency and time delay. In engineering practice, these dynamic parameters of a system can only be determined with a given uncertainty, moreover, parameters can change during operation [14].

There are some methods in the specialist literature for the determination of the robust stability. These compute stability boundaries for a large set of parameters, and the intersection of the stability ranges is determined. These algorithms lead to numerically intensive computations with many limitations, hence the "robust formulation cannot accommodate more than two varying parameters due to increasing model complexity" [15].

In the present study, a perturbation method is presented to determine the robust stability limit, through the turning processes with process damping effect, first. The obtained robust stability limit is insensitive to the fluctuation of the dimensionless time-delay parameter and it forms the lower envelope of the lobe structure.

In Section 3, the computation is applied to a one degree-of-freedom (DoF) turning processes with process damping effect.

In Section 4, the stability chart of the two-cutter turning process is presented to test the method for a multi-DoF system.

2 ROBUST STABILITY COMPUTATION METHOD

2.1 Stability of the turning process

Let us consider the simple 1-DoF model of the orthogonal turning with process damping effect (see Fig.1) [2,16]. The governing equation of the perturbed motion is given by:

$$m\ddot{x}(t)+(c+C_{\mathrm{p}}\tau)\dot{x}(t)+kx(t)=k_1 w(-x(t)+x(t-\tau)), \quad (1)$$

where the parameters are mass m, damping c, process damping coefficient C_p, stiffness k, cutting force coefficient k_1, chip width w, time delay $\tau = 2\pi/\Omega$ and spindle speed Ω.

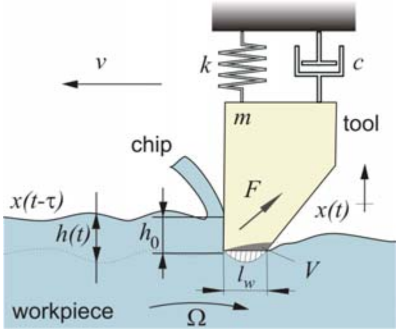

Figure 1: Orthogonal turning model with surface regenerative effect and process damping effect.

The stability boundaries are determined based on the characteristic equation D [3,17], which is found by substituting the trial function $x(t) = Ae^{-\lambda t}$ into Eq. (1) and considering the critical root $\lambda = \mathrm{i}\omega_c$

$$D(\omega_c) = -m\omega_c^{\,2} + \mathrm{i}(c + C_{\mathrm{p}}\tau)\omega_c + k + k_1 w(1 - e^{-\mathrm{i}\omega_c\tau}) = 0. \quad (2)$$

If we consider the spindle speed and the chip width as the two typical parameters of the stability chart (Ω ,w) and the chatter frequency ω_c as independent variable, then a co-dimension 2 problem is defined in the 3 dimensional parameter space based on the real and the imaginary parts of the characteristic equation:

$$\mathrm{Re}(D(\Omega, w, \omega_c)) = 0, \quad (3)$$

$$\mathrm{Im}(D(\Omega, w, \omega_c)) = 0. \quad (4)$$

For a selected damping ratio and process damping coefficient parameters, the resultant stability boundaries (bifurcation lines) can easily be obtained by the MDBM [18,19]. This root-finding method can automatically find multiple boundary curves, even closed ones, that are the stable and unstable islands in the stability charts. The resultant stability diagram is presented in Fig.2a.

2.2 Robust stability

The previous computation method considers the exact model of the mechanical system, and does not take into account the uncertainty of the input parameters, such as natural frequency and spindle speed. The ratio of these parameters determines the dimensionless time delay, which is the main source of the stability problem. Hence, small differences in these parameters could lead to significant change in the lobe structure of the stability chart.

Note, that the uncertainty of the cutting force coefficient k_1 influences the vertical position of the lobes proportionally, while a small uncertainty in process damping coefficient C_p modifies the steepness of the envelope of the lobes slightly, but these cutting force parameters have no influence on the lobe structure.

The complicated lobe structure is generated by the time delay that appears in exponential term in the characteristic equation. If the so-called lobbing-effect is not important, then the computation of the dense lobe structure is not necessary. In this case an extra perturbation ε of the time delayed parameter has to be introduced in the exponential term.

Based on the perturbation method presented in [20], the modified argument of the exponential term has to be considered as an independent variable, which is defined as the regenerative phase shift parameter:

$$\Phi = -\mathrm{i}\omega_c\tau + \varepsilon\,(\mathrm{mod}\ 2\pi). \quad (5)$$

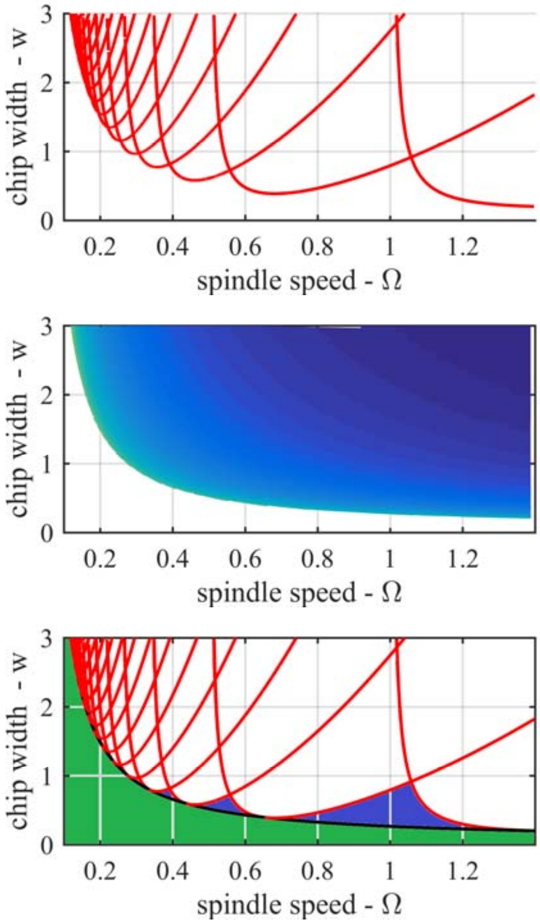

Figure 2: a) Traditional stability lobes. b) Robust stability surface swept by regenerative phase shift parameter c) Robust stability limit (black). The stable area is blue and the robust stable area is green. Dimensionless parameters: m=1, c=0.05, C_p=0.001, k=1, k_1=1.

The characteristic equation reads as

$$D(\Omega, w, \omega_c, \Phi) = \\ -m\omega_c^{\,2} + \mathrm{i}(c + C_{\mathrm{p}}\tau)\omega_c + k + k_1 w(1 - e^{\mathrm{i}\Phi}) = 0. \quad (6)$$

The continuous variation of parameter Φ on the interval [0,2pi] connects the set of boundary curves and forms a surface. The application of the MDBM is essential to solve the resultant co-dimension 2 problem in the extended 4 dimensional parameter space $(\Omega, w, \omega_c, \Phi)$. The resulting surface is plotted in Fig. 2b.

The robust stability limit is defined by the envelope of this surface.

It is shown in [20] that in the vicinity of the parameter points along the envelope the real part of the critical roots λ of the characteristic equation does not change as a function of the perturbation parameter. This extra condition for the robust stability limit is formulated based on the implicit derivation of the characteristic equation as follows (see [20]):

$$\mathrm{Im}\left(\frac{\partial D(\Omega,w,\omega_c,\Phi)}{\partial\omega_c}\overline{\frac{\partial D(\Omega,w,\omega_c,\Phi)}{\partial\Phi}}\right)=0, \qquad (7)$$

Figure 2c shows the traditional stability chart from Fig2a. and the robust stability limit which is determined by the MDBM as a co-dimension 3 problem formed by Eq.(3),(4) and (7) in the extended 4 dimensional parameter space $(\Omega, w, \omega_c, \Phi)$.

This combined chart could be well applied in industrial practice, while the safe range of parameters is presented by the robust stability limit together with the stability pockets of the traditional instability-lobe structure. In these pockets large MRR can be achieved, but one has to be aware of the risk of parameter uncertainty which can affect the position of the lobes.

Note, that during the computation of the robust stability limit there is no need for very high resolution along the spindle speed and the chatter frequency parameter, because the regenerative phase shift parameter eliminates the high sensitivity of the stability chart for these parameters. Thus, the computational time of the MDBM method is similar in case of the traditional stability chart and in the robust stability chart.

3 ROBUST STABILITY LIMIT OF THE TWO-CUTTER TURNING WITH PROCESS DAMPING

In order to present the efficiency of the above presented robust stability computation method, it is applied for the test case of the two-cutter turning process with process damping. The model and the corresponding equation of motion of a two-cutter turning system is given in [21, 22] where the dynamics of the turret is also modelled. The simplified mechanical model is shown in Fig.3. If the effect of process damping [2, 8] is taken into account, too, then an extra damping coefficient appears, which is proportional to the time delay τ and thus the governing equation can be written in the form:

$$\begin{aligned}&\mathbf{M}\ddot{\mathbf{x}}(t)+(\mathbf{C}+C_\mathrm{p}\tau\,\mathbf{E})\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)\\&\qquad=k_1 w(-\mathbf{E}\mathbf{x}(t)+\mathbf{L}\mathbf{x}(t-\tau)),\end{aligned} \qquad (8)$$

Here, $\mathbf{x}$ is the vector of the position coordinates of the tools and the turret (see Fig.3). The coefficient matrices in Eq.(34) are given in Tab.1.

The corresponding characteristic equation for $\lambda=\mathrm{i}\omega_c$ with the regenerative phase shift parameter Φ is given by:

$$\begin{aligned}&D(\Omega,w,\omega_c,\Phi)=\\&\det\left(-\mathbf{M}\omega_c^{\,2}+\mathrm{i}(\mathbf{C}+C_\mathrm{p}\tau\,\mathbf{E})\omega_c+\mathbf{K}+k_1 w(\mathbf{E}-\mathbf{L}e^{\mathrm{i}\Phi})\right)=0\end{aligned} \qquad (8)$$

The resulting stability boundary curves (red) and the robust stability limit (black) are shown in Fig.4.

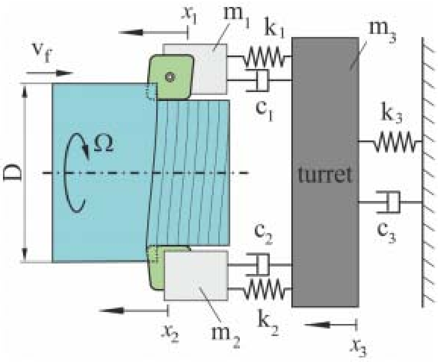

Figure 3: Schematic representation of the two-cutter turning model. Tool 1 and tool 2 are coupled via the cutting force function, too, due to the surface regeneration effect.

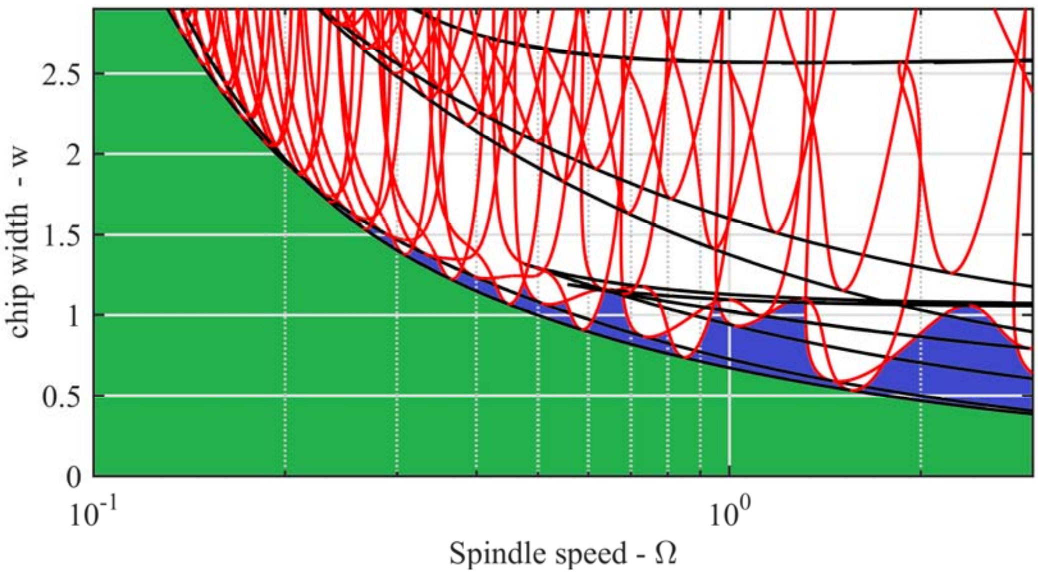

Figure 4: Stability boundaries of the two-cutter turning system (red lines) and its robust stability limits (black lines). Stability chart. The stable area is shaded blue the robust stable area is green. Dimensionless parameters: $m_1 = m_2 = 1$, $m_3 = 10$, $c_1 = c_2 = 0.02$, $c_3 = 0.2$, $k_1 = 1$, $k_2 = 4$, $k_3 = 40$, $C_p = 0.02$, $k = 1$. Note, the logarithmic scale of the spindle speed Ω .

mass matrix	damping matrix	stiffness matrix	self coupling matrix	cross coupling matrix
$\mathbf{M}$	$\mathbf{C}$	$\mathbf{K}$	$\mathbf{E}$	$\mathbf{L}$
$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$	$\begin{bmatrix} c_1 & 0 & -c_1 \\ 0 & c_2 & -c_2 \\ -c_1 & -c_2 & c_1+c_2+c_3 \end{bmatrix}$	$\begin{bmatrix} k_1 & 0 & -k_1 \\ 0 & k_2 & -k_2 \\ -k_1 & -k_2 & k_1+k_2+k_3 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Table 1: The coefficient matrices in the governing equation of the two-cutter turning model.

4 CONCLUSION

In the present study, a perturbation method is presented for calculating the robust stability limit, which forms the lower envelope of the lobe structure.

The method eliminates the essential parameter sensitivity of the stability charts, thus it can be used in case of inaccurate input parameters, such as: natural frequency or spindle speed. In the proposed computation method, a new regenerative phase shift parameter is introduced, which increases the dimension of the parameter space by one, and a complementary equation is also applied.

It is presented, that the Multi-Dimensional Bisection Method can solve the resultant set of equations in the 4 dimensional parameter space efficiently.

Future research is planned to extend this method to milling processes, where the time periodic coefficients in the governing equation have an important effect on the stability behaviour.

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6 REFERENCES

[1] Tlusty, J., Spacek, L., 1954, Self-excited vibrations on machine tools. Prague, Czech Republic: Nakl CSAV. [In Czech.]

[2] Tobias, S.A., 1965, Machine Tool Vibration. Blackie and Son, Ltd., London

[3] Stepan, G., 1989, Retarded Dynamical Systems. Longman, Harlow

[4] Insperger, T., Stepan, G., 2011, Semi-Discretization for Time-Delay Systems, 1 edn. Applied Mathematical Sciences. Springer, 1

[5] Kolmanovskii, V. B., Nosov, V. R., 1986, Stability of functional differential equations. vol. 180, Academic Press, London.

[6] Nayfeh, A. H., Mook, D. T., 1979, Nonlinear Oscillations. John Wiley and Sons, New York.

[7] Khasawneh, F. A., Mann, B. P., Butcher, E. A., 2010, Comparison between collocation methods and spectral element approach for the stability of periodic delay systems. In: 9th IFAC Workshop on Time Delay Systems. 9 (1). pp. 69-74

[8] Altintas, Y., Stepan, G., Merdol, D., Dombovari, Z., 2008, Chatter stability of milling in frequency and discrete time domain. CIRP Journal of Manufacturing Science and Technology. 1 (1). pp. 35-44

[9] Zatarain, M., Dombovari, Z., 2014, Stability analysis of milling with irregular pitch tools by the implicit subspace iteration method. International Journal of Dynamics and Control. 2 (1). pp. 26-34

[10] Bayly, P. V., Halley, J. E., Mann, B. P., Davies, M. A., 2003, Stability of interrupted cutting by temporal finite element analysis. Journal of Manufacturing Science and Engineering. 125 (2). pp. 220-226.

[11] Budak, E., Altintas, Y., 1998, Analytical prediction of chatter stability conditions for multi-degree of systems in milling. Transactions of ASME Journal of Dynamic Systems Measurement and Control. 120. pp. 22-30.

[12] Altintas, Y., Budak, E., 1995, Analytical prediction of stability lobes in milling. CIRP Annals - Manufacturing Technology. 44 (1). pp. 357-362.

[13] Bachrathy, D., Stepan, G., 2013, Improved prediction of stability lobes with extended multi frequency solution. CIRP Annals - Manufacturing Technology. 62 (1). pp. 411-414.

[14] Munoa, J., Zatarain, M., Bediaga, I., 2006, "Determination of the cutting performance and dynamic behaviour of universal milling machines by means of a stability model. In: ISMA2006- International Conference on Noise and Vibration Engineering. Sep 2006, Leuven, Belgium pp. 3789-3804.

[15] Graham, E., Mehrpouya, M., Park, S., 2013, Robust prediction of chatter stability in milling based on the analytical chatter stability. Journal of Manufacturing Processes. 15 (4). pp. 508-517

[16] Eynian, M., Altintas, Y., 2009, Chatter stability of general turning operations with process damping. Journal of Manufacturing Science and Engineering. 131 (4). p. 10.

[17] Niemark, J., 1949, D-subdivision and spaces of quasi-polynomials, Prikl. Mat. Mekh. 13 (4). pp. 349-380. (in Russian)

[18] Bachrathy, D., 2012, Multi-dimensional bisection method (mdbm) - fast computation of all the roots of a non-linear function for higher parameter dimension and co-dimension..

[19] Bachrathy, D., Stepan, G., 2012, Bisection method in higher dimensions and the efficiency number. Periodica Polytechnica Mechanical Engineering. 56 (2). pp. 81-86.

[20] Bachrathy, D., 2015, Robust stability limit of delayed dynamical systems. Periodica Polytechnica. Engineering. Mechanical Engineering, 59(2), 74

[21] Reith, M.J., Bachrathy, D., Stepan, G., 2014, Comparing stability and dynamic behaviour of different multi-cutter turning models. In: ENOC 2014, Wien,, 494, p. 2

[22] Reith, M.J., Stepan, G., 2016, Exploitation of non-proportional damping in machine tools for chatter

suppression. Journal Mechanism and Machine Theory submitted