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STATE-DEPENDENT, NON-SMOOTH MODEL OF CHATTER VIBRATIONS IN TURNING

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ABSTRACT

This paper deals with the modeling and analysis of the cutting tool's global dynamics in the orthogonal cutting process of turning operations considering the effect of state dependency and fly-over in one model. In particular, the one-degree-of-freedom non-smooth model, presented by Wahi and Chatterjee in 2008, is extended by the consideration of vibrations in the direction perpendicular to the feed velocity. This results in the state-dependency of the model and gives an additional direction in which fly-over can occur. The constructed mathematical model consists of a nonlinear PDE, which describes the evolution of the surface height of the workpiece and a two-degree-of-freedom ODE, which governs the motion of the tool. The PDE is connected to the solution of the ODE by a non-local, non-smooth boundary condition. For the case when the tool is within the cut, this model gives the conventional model of turning governed by delay-differential equations with state-dependent delays. In order to study the effect of vibrations in the tangential direction numerical simulations are carried out and their results are compared to the model presented by Wahi and Chatterjee (2008).

1 INTRODUCTION

Machine tool chatter is the large amplitude vibration of the cutting tool in machining operations involving intermittent loss of contact between the tool and the workpiece. These vibrations

are highly deteriorative to the machining process since they result in poor workpiece surface quality and machining accuracy and, at the same time, they also increase the tool wear or even damage the tool. Consequently, machine tool chatter limits the material removal rate in addition to the limitations given by the power supply of the machine [1,2]. Therefore, the elimination of chatter would lead to a drastic increase in productivity. Although there exist methods to suppress vibrations during machining operations (see e.g. [3,4]), these techniques do not solve the problem of chatter completely. As a result, machine tool chatter is still a crucial issue in manufacturing.

In the recent decades several articles have been published on the modeling of machine tool chatter. These articles mainly study turning and milling operations, however many of them investigate drilling and grinding operations, too. The presented models relate machine tool chatter to friction [5], to thermo-mechanical effects [6] and to the so called surface regeneration [7]. Out of these chatter sources regenerative chatter is the most important [8,9].

This particular paper deals with the modeling of regenerative chatter in turning processes. The essence of the regenerative phenomenon is that the cutting force depends on the chip thickness, which can be calculated as the difference between the actual position of the cutting tool and its position one revolution before. Therefore, the governing equation of the tool involves delayed values of the state. Depending on the selection of machining parameters, these delayed terms can lead to the loss of

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stability which is associated with chatter vibrations. Due to the presence of delayed terms, the majority of mathematical models represent the regenerative phenomenon by delay-differential equations (DDEs). For turning, mechanical models resulting in smooth DDEs assume that the cutting tool remains in the cut at all time. The cutting force is defined either by Taylor's or Tobias's law, which both describe dependence on the chip thickness and thus dependence on some delayed state of the tool. When the local stability of the cutting process is studied then smooth DDE models can be verified since the cutting edge has permanent contact with the workpiece in case of stationary cutting. However, chatter itself is defined as the motion where the tool repeatedly loses and re-establishes contact with the workpiece. Therefore, when one is interested in not only the local but also in the global behaviour of the cutting tool, then the standard smooth DDE models have to be modified by the consideration of the so called fly-over of the tool. The term *fly-over* refers to the tool's motion for the case when it is out of cut. During fly-over the tool vibrates freely and the delayed terms disappear from the governing equations which results the non-smoothness of the mathematical model [10].

The first models, dealing with the global dynamics of chatter in turning, were carried out by Wahi and Chatterjee [11] and Dombovari et al [12]. Both models neglected the tool's vibration in the direction perpendicular to the feed velocity (which coincides with the tangential direction of the workpiece) and extended the conventional, smooth DDE model by modeling the surface height of the workpiece. This paper deals with the extension of the model presented in [11] by the consideration of the tool's motion in the tangential direction of the workpiece. Moreover, the relation is shown between different models for turning and the effect of the tool's vibration in the tangential direction is presented by the comparison of time-domain simulations for the herein introduced model and the model presented in [11]. The structure of the paper is the following. First, the most important models are described for regenerative chatter in turning. Then the proposed model is introduced. Thereafter relation between different models is discussed and time-domain simulations are presented. Finally some conclusions are drawn from the results.

2 REGENERATIVE CHATTER MODELS OF TURNING

As mentioned above, the existing models of turning can be categorized based on whether they consider the fly-over effect or not. Those considering the fly-over of the tool, result in non-smooth dynamical systems while the ones neglecting this phenomenon give smooth DDEs.

2.1 Smooth models

Models which assume permanent contact between the tool and the workpiece throughout the machining process are de-

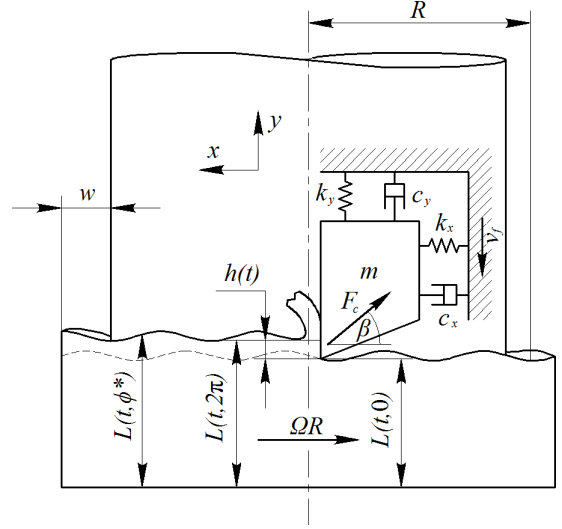


FIGURE 1. Mechanical model of the orthogonal cutting process

scribed by smooth DDEs. If the spindle speed is constant then, depending on whether tangential vibrations are considered or not the delay is state dependent or constant, respectively.

2.1.1 Model with constant delay Figure 1 shows the simplest model of the orthogonal cutting process. In particular, the cutting tool is substituted by a spring-mass system which is carried by a frame moving with a constant feed velocity magnitude v_f in direction $-y$, normal to the removed surface and parallel to the axis of the workpiece. The (x, y, z) coordinate system is fixed to the moving frame (see Figure 2). For simplicity, it is assumed that the modal matrices of the tool, corresponding to its first eigenfrequencies in the normal direction y and tangential direction x , are diagonal thus there is no coupling between x and y directions. The non-zero elements of the modal mass matrices are denoted by m . The elements of the modal stiffness matrix are k_x and k_y , while the elements of the modal damping matrix are c_x and c_y in the corresponding directions. The magnitude of the cutting force is denoted by F_c , while the angle between the tangential direction and the cutting force is given by β . When the tool is considered to be rigid in the tangential direction (that is $k_x \rightarrow \infty$) then the dynamics of the tool is governed by

$$m\ddot{y}(t) + c_y\dot{y}(t) + k_y y(t) = \sin(\beta)F_c(t). \quad (1)$$

Throughout this paper the Taylor cutting force model is used, therefore the cutting force is calculated as

$$F_c(t) = Kw h^q(t), \quad (2)$$

where K is called cutting force coefficient, w is the chip width of the tool, q is the cutting force exponent and h is the chip thickness, calculated as

$$h(t) = y(t - \tau) - y(t) + v_f \tau. \quad (3)$$

The time-length of the last complete rotation at time t is denoted by τ . It is assumed that the spindle speed is constant and the tool is rigid in direction x , therefore the delay can be calculated as $\tau = 2\pi/\Omega$. (Details on Tobias's cutting force law can be found e.g. in [13].)

By introducing dimensionless time $\tilde{t} = \omega_y t$ and dimensionless delay $\tilde{\tau} = \omega_y \tau$ and dropping the tilde immediately, equations (1)–(3) give

$$\ddot{y}(t) + 2\zeta_y \dot{y}(t) + y(t) = \kappa_y (y(t - \tau) - y(t) + v_f^* \tau)^q, \quad (4)$$

where $v_f^* = v_f/\omega_y$ and $\kappa_y = K_y w/(m\omega_y^2)$, with $K_y = \sin(\beta)K$ being the cutting force coefficient in direction y . The natural angular frequency and damping ratio in direction y are $\omega_y = \sqrt{k_y/m}$ and $\zeta_y = c_y/(m\omega_y)$, respectively. The trivial equilibrium of (4) is $y_e = \kappa_y (v_f^* \tau)^q$ which gives the desired nominal chip thickness $h_0 = v_f^* \tau$. By introducing dimensionless coordinate $\tilde{y} = y/h_0$ and dropping the tilde immediately, one obtains

$$\ddot{y}(t) + 2\zeta_y \dot{y}(t) + y(t) = \hat{\kappa}_y (y(t - \tau) - y(t) + 1)^q, \quad (5)$$

where $\hat{\kappa}_y = \kappa_y h_0^{q-1}$. This is a nonlinear DDE with one constant delay. It has a well understood behavior since its stability and local nonlinear dynamics have been analyzed by several authors (see, e.g., Chapter 5.1.2 in [14] and [15]). The structure of the stability chart on the (Ω, w) parameter plane and the existence of subcritical bifurcation around the stationary motion have been verified by measurements also (see e.g. [7]).

2.1.2 SD-DDE model When the tool is not rigid in the tangential direction, then an additional equation appears in the form

$$m\ddot{x}(t) + c_x \dot{x}(t) + k_x x(t) = -\cos(\beta) F_c(t). \quad (6)$$

For this equation, the same way as for (1), the introduction of dimensionless time and dimensionless coordinate gives

$$\ddot{x}(t) + \mu \zeta_x \dot{x}(t) + \mu^2 x(t) = -\hat{\kappa}_x (y(t - \tau) - y(t) + 1)^q. \quad (7)$$

where $\hat{\kappa}_x = K_x w h_0^{q-1}/(m\omega_y^2)$ with $K_x = \cos(\beta)K$ being the cutting force coefficient in direction x . The damping ratio in direction x is denoted by $\zeta_x = c_x/(m\omega_x)$, while $\mu = \sqrt{k_x/k_y}$ is the

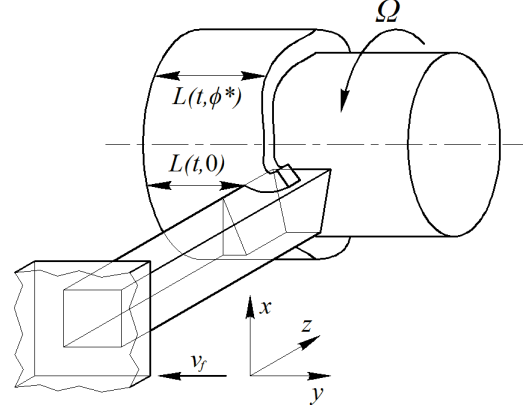


FIGURE 2. 3D sketch of the mechanical model

stiffness ratio. Together, (5) and (7) determine the motion of the tool however, due to the flexibility of the tool in direction x , the delay is not constant but given implicitly by

$$\int_{t-\tau}^t \hat{R} \hat{\Omega}(s) ds = 2\hat{R}\pi - x(t) + x(t - \tau), \quad (8)$$

where $\hat{R} = R/h_0$ is the dimensionless radius of the workpiece (see Figure 1) and $\hat{\Omega} = \Omega/\omega_y$ is the dimensionless spindle speed. Note, that because of (8) the delay depends on state variable x . For more details on the SD-DDE model see Section 5.1.1 in [14], for results on its stability see [16], and for its local nonlinear dynamics see [17].

2.2 Non-smooth models

Note, that the above equations are valid only for the case when $h(t) \geq 0 \forall t$, since the cutting force cannot be negative at any time. Therefore, the above models can describe the local dynamics of the tool, only. When the global dynamics of the tool is considered then the governing equations are inevitably non-smooth. This property is due to the fly-over of the tool which results that the force term F_c becomes zero for $h < 0$. However, the application of solely this condition to the above equations would not lead to a correct model. This is owing to the fact that uncut segments reappear in the calculation of the chip thickness after one or more revolutions. Therefore, in addition to the consideration of the non-smoothness of the force term one also has to describe the evolution of the workpiece surface in direction y .

Models describing the global dynamics of the cutting tool were carried out in [12] and in [11]. Both models assume constant spindle speed and the tool to be rigid in the tangential direction. Furthermore, they introduce a surface height function (denoted by L in Figures 1 and 2), which describes the evolution of the workpiece surface in direction y . The difference between

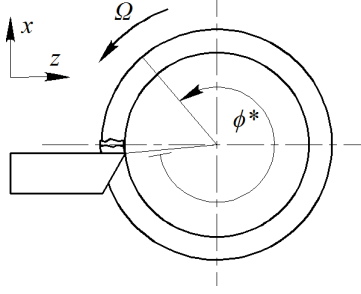


FIGURE 3. Mechanical model in the (x, z) plane

these models is that while [12] calculates the value of the surface height from an algebraic equation, [11] determines its value from a PDE. In the following, the models presented in [12] and [11] are briefly described.

2.2.1 DDAE model (Dombovari et al.) By the consideration of the non-smoothness of the cutting force (5) is modified as

$$\ddot{y}(t) + 2\zeta_y \dot{y}(t) + y(t) = \begin{cases} \hat{\kappa}_y (\bar{L}(t - \tau) - y(t) + 1)^q & \text{if } \bar{L}(t - \tau) - y(t) + 1 \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

where the evolution of the dimensionless surface height function is defined by

$$\bar{L}(t) = \begin{cases} y(t) & \text{if } \bar{L}(t - \tau) - y(t) + 1 \geq 0 \\ \bar{L}(t - \tau) + 1 & \text{otherwise.} \end{cases} \quad (10)$$

It can be seen from the definition of \bar{L} that when fly-over occurs then the surface height in the next revolution simply becomes the actual "skipped" surface height plus the nominal chip thickness. When the tool is in the cut, then the chip thickness is calculated as usual. Equations (9) and (10) define a non-smooth delay-differential algebraic equation (DDAE), where (10) is the algebraic, while (9) is the delay-differential part, both being non-smooth.

2.2.2 Simple PDE-ODE model (Wahi and Chatterjee) Note that in the DDAE model, function $\bar{L}(t)$ defines the dimensionless surface height at time t for the angular position where the tool tip is located, while the coordinate $\bar{L} = 0$ is fixed to the moving frame of the tool. On the other hand the simple PDE-ODE model describes the dimensionless surface height at

time t not only for one point, but for the whole cylindrical surface of the workpiece, while the coordinate $L = 0$ is fixed to a steady coordinate system parallel to y . That is the simple PDE-ODE model defines L as a bivariate function $L(t, \phi)$ with domains $t \geq 0$, $\phi \in [0, 2\pi]$ (see Figures 1 and 2). Coordinate $\phi = 0$ is fixed to the angular position of the tool tip and the positive direction of coordinate ϕ coincides with the direction of rotation (see Figure 3). Utilizing that the surface height does not change along the domain $\phi \in (0, 2\pi]$, one can derive a hyperbolic PDE (its derivation is detailed in the next section) in the form

$$\frac{\partial L(t, \phi)}{\partial t} = -\hat{\Omega} \frac{\partial L(t, \phi)}{\partial \phi}, \quad \forall t, \phi \in (0, 2\pi]. \quad (11)$$

At the angular position $\phi = 0$ a non-smooth non-local boundary condition is given in the form

$$L(t, 0) = \begin{cases} L(0, 0) - \hat{v}_f t + y(t) & \text{if } h(t) \geq 0 \\ L(t, 2\pi) & \text{otherwise} \end{cases} \quad (12)$$

where $\hat{v}_f = v_f^*/h_0$ is the dimensionless feed velocity and it is assumed, that cutting starts at $t = 0$, therefore $L(0, \phi)$, $\phi \in (0, 2\pi]$ is the initial surface height function. The tool's motion is determined by the non-smooth ODE

$$\ddot{y}(t) + 2\zeta_y \dot{y}(t) + y(t) = \begin{cases} \hat{\kappa}_y h^q(t) & \text{if } h(t) \geq 0 \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

The surface height difference on the cutting edge gives the chip thickness, that is

$$L(t, 2\pi) - L(t, 0) = h(t). \quad (14)$$

When the tool is in the cut ($h(t) \geq 0$) then the chip thickness is given by

$$h(t) = L(t, 2\pi) - L(0, 0) + \hat{v}_f t - y(t), \quad (15)$$

otherwise the chip thickness is zero, thus

$$h(t) = 0. \quad (16)$$

Note that this PDE-ODE description is more general than the DDAE since the modelled physical process is a contact problem of two rigid bodies, which both can be described by PDEs connected with non-smooth boundary conditions. Since it was proposed in [11] the simple PDE-ODE model has been investigated by other authors as well (see eg. [18]).

3 EXTENDED PDE-ODE MODEL

As it was mentioned in the previous section, existing models for the description of global dynamics in turning consider vibrations only in direction y and the tool is therefore assumed to be rigid in direction x . However, in real cases k_x and k_y are usually in the same order of magnitude, hence, when the tool leaves the workpiece then it starts considerable vibrations in both x and y directions. This makes the extension of existing models in direction x reasonable. In this section the derivation of the governing equation of such extended model is detailed.

Let us use the same coordinate system as described for the simple PDE-ODE model. Consider a particular material point on the surface of the workpiece. If this material point is within the domain $\phi \in (0, 2\pi]$, then the corresponding surface height does not change. This results that if $\phi = \phi(t)$ is a function describing the relative angular position between the material point and the tool tip then the following relation holds

$$L(t, \phi(t)) = L(t + \Delta t, \phi(t + \Delta t)) \quad \forall t, \forall \Delta t, \phi \in (0, 2\pi]. \quad (17)$$

It can be assumed, that L is continuous in both variables on the above given domains, therefore

$$\lim_{\Delta t \rightarrow 0} \frac{L(t + \Delta t, \phi(t + \Delta t)) - L(t, \phi(t))}{\Delta t} = 0, \quad \forall t, \phi \in (0, 2\pi]. \quad (18)$$

which, at the limit gives

$$\frac{dL(t, \phi(t))}{dt} = \frac{\partial L}{\partial t} + \frac{\partial L}{\partial \phi} \frac{d\phi}{dt} = 0, \quad \forall t, \phi \in (0, 2\pi]. \quad (19)$$

Since the coordinate $\phi = 0$ is fixed to the tool tip, the relative angular displacement between the tool and the workpiece surface can be approximated as

$$\phi(t) \approx \frac{x(t)}{\hat{R}} + \varphi(t) \quad (20)$$

if the ratio $\sup\{|x(t)|\}/\hat{R}$ is small enough. Here $\varphi(t)$ is the angular displacement of the spindle. Equation (19) therefore gives

$$\frac{\partial L(t, \phi)}{\partial t} = - \left(\frac{\dot{x}(t)}{\hat{R}} + \dot{\Omega}(t) \right) \frac{\partial L(t, \phi)}{\partial \phi} \quad \forall t, \phi \in (0, 2\pi], \quad (21)$$

where $\dot{\Omega}(t) = (1/\omega_y)(d\varphi(t)/dt)$. This PDE governs the evolution of the dimensionless surface height function. The chip thickness is given again by (15) or (16) depending on whether the tool is in or out of the cut, respectively. Therefore, the boundary

conditions will be the same as in case of the simple PDE-ODE model except for the switching conditions. The switching conditions will differ since now the tool can leave the cut in direction x as well. This happens when $\dot{\phi}(t) < 0$, that is when the relative motion between the tool and the workpiece changes its direction. Therefore, both $\dot{\phi}(t) \geq 0$ and $h(t) \geq 0$ have to be satisfied for the tool to be in the cut. Hence, the boundary conditions read as

$$L(t, 0) = \begin{cases} L(0, 0) - \hat{v}_f t + y(t) & \text{if } h(t) \geq 0 \text{ and } \dot{\phi}(t) \geq 0 \\ L(t, 2\pi) & \text{otherwise.} \end{cases} \quad (22)$$

The tool's motion is now described by the non-smooth system of ODEs

$$\ddot{y}(t) + 2\zeta_y \dot{y}(t) + y(t) = \begin{cases} \hat{\kappa}_y h^q(t) & \text{if } h(t) \geq 0 \text{ and } \dot{\phi}(t) \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

$$\ddot{x}(t) + 2\mu \zeta_x \dot{x}(t) + \mu^2 x(t) = \begin{cases} \hat{\kappa}_x h^q(t) & \text{if } h(t) \geq 0 \text{ and } \dot{\phi}(t) \geq 0 \\ 0 & \text{otherwise.} \end{cases} \quad (24)$$

Equations (21), (23) and (24) define the PDE-ODE system subject to the boundary condition (22). The PDE is coupled nonlinearly to the ODEs since the term $\dot{x}(t)$ shows up in (21) as a multiplier. The PDE and the ODEs are also coupled via the boundary condition (22). Note that the ODEs are already two-degree-of-freedom approximations obtained by modal measurements of the tool. This model can be easily extended by the consideration of arbitrary number of modes, which would increase the dimension of the ODE system.

4 RELATION AMONG DIFFERENT MODELS

In this section, the relation is studied between the extended PDE-ODE model and the above described models taken from the literature.

Clearly, if the spindle speed is constant, that is, $\varphi(t) = \Omega t$, and the vibrations in direction x are neglected then the extended PDE-ODE model is exactly the same as the simple PDE-ODE model.

Define τ as the time necessary for one complete rotation with respect to the tool tip, that is

$$\phi(t) - \phi(t - \tau) = 2\pi. \quad (25)$$

Due to (17), this gives

$$L(t - \tau, 0) = L(t, 2\pi) \quad \forall t. \quad (26)$$

Therefore, if the tool is in the cut, then from (14) and (22) the chip thickness is

$$h(t) = L(t - \tau, 0) - L(t, 0) = y(t - \tau) - y(t) + \hat{v}_f \tau. \quad (27)$$

Since $\hat{v}_f \tau = 1$, by the substitution of (27) to (23) and (24) one obtains (5) and (7), respectively. Note, also that (25) is equivalent to (8) after the substitution of (20). Hence, if the tool is permanently in the cut, then the solution of the extended PDE-ODE model at angular position $\phi = 0$ gives the SD-DDE model.

When vibrations in direction x are neglected and the spindle speed is constant then the SD-DDE model gives the DDE model with constant delay, therefore under these assumptions the extended PDE-ODE model also gives the DDE model with constant delay when $h \geq 0$ for all t .

Let us transform now the dimensionless surface height function to the moving frame as

$$\tilde{L}(t, \phi) = L(t, \phi) + L(0, 0) - \hat{v}_f t. \quad (28)$$

Now if $\bar{L}(t) := \tilde{L}(t, 0)$, then the substitution of (26) to (15) and (16) gives the corresponding cases in (10). Therefore, for constant spindle speed and under the neglect of vibrations in direction x , (23) gives (9), that is, at angular position $\phi = 0$ the extended PDE-ODE model gives the DDAE model. Note, that with (28), the simple PDE-ODE model at angular position $\phi = 0$ also gives precisely the DDAE model.

According to the above discussed reasons the presented models from the literature are special cases of the extended PDE-ODE model.

5 CALCULATIONS

In this section some results are shown for the extended PDE-ODE model and the simple PDE-ODE model in the form of numerical simulations. First, the numerical method, which was used for the calculation of the results is detailed, then some results are presented and discussed.

5.1 Numerical scheme

In order to carry out simulations for the dynamical system defined by (21)-(24) one first have to discretize the PDE given by (21). This discretization is done by using central differences for which the idea was taken from [19]. At time instant t , this method represents the function $L(t, \phi)$ by an $(n+1)$ -dimensional vector of distinct points of the state function, corresponding to the nodes of an equidistant mesh on $\phi \in (0, 2\pi]$. The state function is therefore discretized in its second variable as

$$L_i(t) = L(t, \phi_i), \quad \phi_i = \frac{2\pi(i-1)}{n}, \quad i = 1, 2, \dots, n+1. \quad (29)$$

Using these state values the derivative with respect to ϕ is approximated by central differences as

$$L'_i(t) = \left. \frac{\partial L(t, \phi)}{\partial \phi} \right|_{\phi=\phi_i} \approx \begin{cases} \frac{L_{i+1}(t) - L_i(t)}{\Delta\phi} & i = 1 \\ \frac{L_{i+1}(t) - L_{i-1}(t)}{2\Delta\phi} & i = 2, 3, \dots, n \\ \frac{L_i(t) - L_{i-1}(t)}{\Delta\phi} & i = n+1. \end{cases} \quad (30)$$

Utilizing this discretization, PDE (21) can be approximated by a system of $n+1$ number of ODEs in the form

$$\dot{\mathbf{L}}(t) = - \left(\hat{\Omega}(t) + \frac{\dot{x}(t)}{\hat{R}} \right) \mathbf{L}'(t) = \mathbf{M}(t, \dot{x}(t)) \mathbf{L}(t), \quad (31)$$

where $\mathbf{L}(t)$ and $\mathbf{L}'(t)$ are vectors of $L_i(t)$ and $L'_i(t)$, respectively, and

$$\mathbf{M}(t, \dot{x}(t)) = - \left(\hat{\Omega}(t) + \frac{\dot{x}(t)}{\hat{R}} \right) \begin{bmatrix} -\frac{1}{\Delta\phi} & \frac{1}{\Delta\phi} & & 0 \\ -\frac{1}{2\Delta\phi} & 0 & \frac{1}{2\Delta\phi} & \\ & \ddots & \ddots & \ddots \\ 0 & & -\frac{1}{2\Delta\phi} & 0 & \frac{1}{2\Delta\phi} \\ & & & -\frac{1}{\Delta\phi} & \frac{1}{\Delta\phi} \end{bmatrix} \quad (32)$$

is an $(n+1) \times (n+1)$ tridiagonal matrix. After the introduction of vectors

$$\mathbf{z}_x = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}, \quad \mathbf{f}_x(t, y(t), L_{n+1}(t)) = \begin{bmatrix} 0 \\ -\hat{\kappa}_x h(t)^q \end{bmatrix}, \quad (33)$$

$$\mathbf{z}_y = \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix}, \quad \mathbf{f}_y(t, y(t), L_{n+1}(t)) = \begin{bmatrix} 0 \\ \hat{\kappa}_y h(t)^q \end{bmatrix}, \quad (34)$$

matrices

$$\mathbf{A}_x = \begin{bmatrix} 0 & 1 \\ -\mu^2 & -2\mu\zeta_x \end{bmatrix}, \quad \mathbf{A}_y = \begin{bmatrix} 0 & 1 \\ -1 & -2\zeta_y \end{bmatrix} \quad (35)$$

and dimensionless chip thickness $h(t) = L_{n+1}(t) - L_1(0) + \hat{v}_f t - y(t)$, the possible tool motions can be summarized as follows.

- **Cutting:** $h(t) \geq 0$ and $\left(\hat{\Omega}(t) + \frac{\dot{x}(t)}{\hat{R}} \right) \geq 0$

The tool's motion is governed by

$$\dot{\mathbf{z}}_x(t) = \mathbf{A}_x \mathbf{z}_x(t) + \mathbf{f}_x(t, y(t), L_{n+1}(t)), \quad (36)$$

$$\dot{\mathbf{z}}_y(t) = \mathbf{A}_y \mathbf{z}_y(t) + \mathbf{f}_y(t, y(t), L_{n+1}(t)), \quad (37)$$

while the boundary condition is enforced by

$$L_1(t) = L_1(0) - \hat{v}_f + y(t). \quad (38)$$

• **Fly-over:** $h(t) < 0$ or $\left(\hat{\Omega}(t) + \frac{\dot{x}(t)}{\hat{R}}\right) < 0$

The tool's motion is governed by

$$\dot{\mathbf{z}}_x(t) = \mathbf{A}_x \mathbf{z}_x(t), \quad (39)$$

$$\dot{\mathbf{z}}_y(t) = \mathbf{A}_y \mathbf{z}_y(t), \quad (40)$$

while the boundary condition is enforced by

$$L_1(t) = L_{n+1}(t). \quad (41)$$

If the initial function $L(0, \phi)$ and the initial state $(\mathbf{z}_x(0), \mathbf{z}_y(0))$ of the tool are given then the solution of the approximate system can be determined. For this purpose the 4th-order Runge-Kutta method was used. Note that the accuracy of the solution is sensitive to the accurate determination of the switching points. Hence, close to the switching points smaller time steps should be used. For the herein presented numerical solutions the accurate determination of switching points was done by an interval-halving technique.

5.2 Comparison

The simulations, shown in Figures 4 and 5, were carried out using the parameter set of Table 1. These results were obtained by the built-in ode23 solver of Matlab. For the detection of switching points the built-in event location option of the solver was used. The resolution of the finite dimensional approximation of the PDE was $n = 500$, while the relative and absolute tolerance of the solver was set to 10^{-8} and 10^{-6} , respectively. It can be seen in Figure 4, that considerable vibrations of the tool are present in direction x although the tool is stiffer in this direction ($k_x = 4k_y$). It can also be observed that the vibration of the tool in direction x does not affect the vibrations in direction y . This is due to the fact that for the simulation shown in Figure 4 the velocity of the tool does not exceed the limit where the contact would be lost (i.e., $\dot{\phi}(t) \geq 0$). Note that the ratio $\rho = v_f/(R\Omega)$ of the feed velocity and the cutting speed is also small which resulted very small differences in stability between the smooth models with state-dependent and constant delays (see Figure 3 in [16]). Also note that in practice directions x and y are coupled, which would result differences in the results even if $\dot{\phi}(t) < 0$.

6 CONCLUSIONS

This paper investigated the modeling of the global dynamics of the cutting tool in turning operations. A new model was

TABLE 1. Used parameters for the numerical simulation

$\zeta_x = \zeta_y$	$\hat{k}_x = \hat{k}_y$	μ	q	$\hat{\Omega}$	\hat{v}_f	\hat{R}
0.05	2	2	3/4	2	2	100

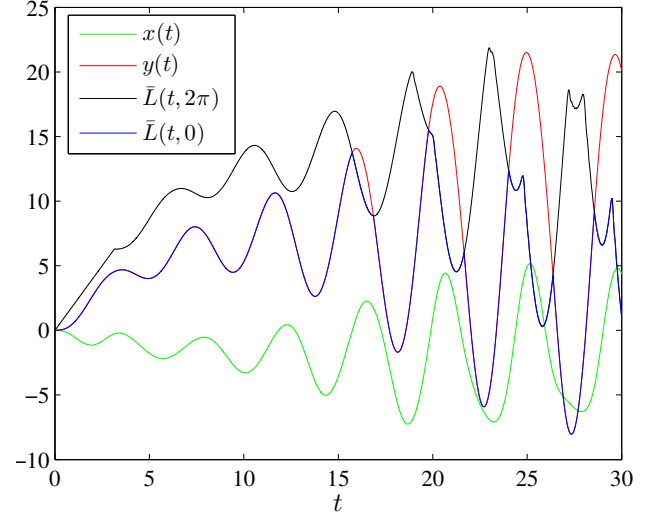


FIGURE 4. Time domain simulation of the extended PDE-ODE model

presented which considered the non-smoothness given by the fly-over of the tool. It was shown that this new model is more general than the existing ones, and the models taken from the literature can be obtained as special cases of the herein presented model. A method for the discretization of this PDE-ODE problem was described, then it was shown by simulations that the tool is subject to considerable vibrations in the direction perpendicular to the feed.

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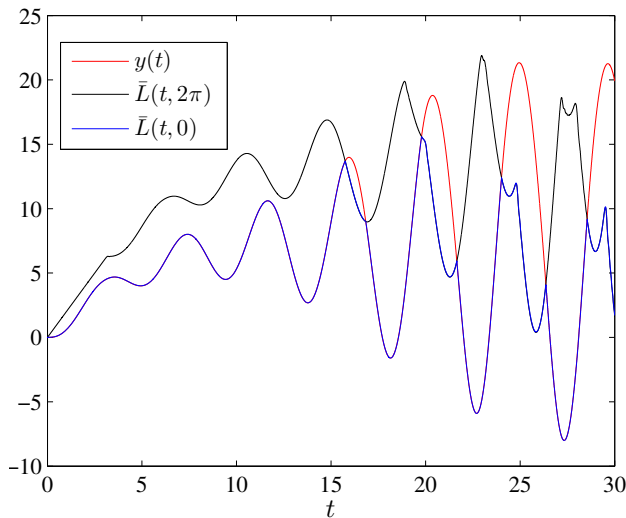


FIGURE 5. Time domain simulation of the simple PDE-ODE model

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